20311 Statistics for Business and Economics II

Homework No. 5

Individual homework

11.14, 11.27, 11.28 part (a) and (b), 11.36 part (a), (b), (c), (e)

11.14 (a) To test at the 0.05 level of significance whether there is any evidence of a difference in the mean distance traveled by the golf balls differing in design, you conduct an F test:

\[ H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \quad \text{vs} \quad H_1: \text{At least one mean is different.} \]

Decision rule: \( df: 3, 36. \) If \( F_{\text{STAT}} > 2.866, \) reject \( H_0. \)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>2990.99</td>
<td>3</td>
<td>996.9966</td>
<td>53.02982</td>
<td>2.73E-13</td>
<td>2.866265</td>
</tr>
<tr>
<td>Within Groups</td>
<td>676.8244</td>
<td>36</td>
<td>18.80068</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3667.814</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since \( F_{\text{STAT}} = 53.03 \) is greater than the critical bound of 2.866, reject \( H_0. \) There is enough evidence to conclude that there is significant difference in the mean distance traveled by the golf balls differing in design.

Note: The critical bound of \( F \) is obtained using Excel. The critical bound of \( F \) using the Table in the text with 3 numerator and 30 denominator degrees of freedom is 2.92.

(b) To determine which of the means are significantly different from one another, you use the Tukey-Kramer procedure to establish the critical range: \( Q_\alpha = 3.79 \)

\[
\text{critical range} = Q_\alpha \sqrt{\frac{MSW}{2} \left( \frac{1}{n_j} + \frac{1}{n_j} \right)} = 3.79 \sqrt{\frac{18.8007}{2} \left( \frac{1}{10} + \frac{1}{10} \right)} = 5.1967
\]

PHStat output:

<table>
<thead>
<tr>
<th>Tukey Kramer Multiple Comparisons</th>
<th>Sample</th>
<th>Sample</th>
<th>Comparison</th>
<th>Absolute Difference</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1 Mean Size Comparison</td>
<td>Group 1 to Group 2</td>
<td>11.902</td>
<td>Means are different</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 206.614 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 218.516 10</td>
<td>Group 1 to Group 3</td>
<td>19.974</td>
<td>Means are different</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 226.588 10</td>
<td>Group 1 to Group 4</td>
<td>22.008</td>
<td>Means are different</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 228.622 10</td>
<td>Group 2 to Group 3</td>
<td>8.072</td>
<td>Means are different</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group 2 to Group 4</td>
<td>10.106</td>
<td>Means are different</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSW 18.800677</td>
<td>Group 3 to Group 4</td>
<td>2.034</td>
<td>Means are not different</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
At 5% level of significance, there is enough evidence to conclude that mean traveling distances between all pairs of designs are different with the only exception of the pair between design 3 and design 4.

(c) The assumptions needed in (a) are (i) the samples are randomly and independently drawn, (ii) populations are normally distributed, and (iii) populations have equal variances.

(e) In order to produce golf balls with the farthest traveling distance, either design 3 or 4 can be used.

11.27 To test at the 0.01 level of significance whether there is any difference in the mean thickness of the wafers for the five positions, you conduct an $F$ test:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

where $1 = \text{position 1}$, $2 = \text{position 2}$, $3 = \text{position 18}$, $4 = \text{position 19}$, $5 = \text{position 28}$

$$H_1: \text{At least one mean is different.}$$

Decision rule: $df$: 4, 116. If $F_{STAT} > 3.4852$, reject $H_0$.

\[
\text{ANOVA}
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Source of Variation} & SS & df & MS & F & P-value \\
\hline
\text{Rows} & 601.5 & 29 & 20.74138 & 5.922219 & 1.93E-12 \\
\text{Columns} & 1417.733 & 4 & 354.4333 & 101.2002 & 6.84E-37 \\
\text{Error} & 406.2667 & 116 & 3.502299 & & \\
\hline
\text{Total} & 2425.5 & 149 & & & \\
\hline
\end{array}
\]

Test statistic: $F_{STAT} = 101.2$

Decision: Since $F_{STAT} = 101.2$ is greater than the critical bound of 3.4852, reject $H_0$. There is enough evidence to conclude that the means of the thickness of the wafers are different across the five positions.

To determine which of the means are significantly different from one another, you use the Tukey-Kramer procedure to establish the critical range:

$$Q_\alpha = 4.71$$

$$\text{critical range} = Q_\alpha \sqrt{\frac{MSE}{r}} = 4.71 \sqrt{\frac{3.5023}{30}} = 1.609$$

Pairs of means that differ at the 0.01 level are marked with * below.

$$|\bar{X}_1 - \bar{X}_2| = 2.2* \quad |\bar{X}_1 - \bar{X}_3| = 5.533* \quad |\bar{X}_1 - \bar{X}_4| = 8.567*$$

$$|\bar{X}_1 - \bar{X}_5| = 6.533* \quad |\bar{X}_2 - \bar{X}_3| = 3.333* \quad |\bar{X}_2 - \bar{X}_4| = 6.367*$$

$$|\bar{X}_2 - \bar{X}_5| = 4.333* \quad |\bar{X}_3 - \bar{X}_4| = 3.033* \quad |\bar{X}_3 - \bar{X}_5| = 1$$

$$|\bar{X}_4 - \bar{X}_5| = 2.033*$$

At 1% level of significance, the $F$ test concludes that there are significant differences in the mean thickness of the wafers among the 5 positions. The Tukey-Kramer multiple comparison test reveals that the mean thickness between all the pairs are significantly different with only the exception of the pair between position 18 and position 28.
11.28 (a) \( H_0: \mu_1 = \mu_2 = \mu_3 \) where 1 = 2 days, 2 = 7 days, 3 = 28 days

\( H_1: \) At least one mean differs.

Decision rule: If \( F_{\text{STAT}} > 3.114 \), reject \( H_0 \).

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Source of Variation} & \text{SS} & \text{df} & \text{MS} & \text{F} & \text{P-value} & \text{F crit} \\
\hline
\text{Rows} & 21.17006 & 39 & 0.542822 & 5.752312 & 2.92E-11 & 1.553239 \\
\text{Columns} & 50.62835 & 2 & 25.31417 & 268.2556 & 1.09E-35 & 3.113797 \\
\text{Error} & 7.360538 & 78 & 0.094366 & & & \\
\text{Total} & 79.15894 & 119 & & & & \\
\hline
\end{array}
\]

Test statistic: \( F = 268.26 \)

Decision: Since \( F_{\text{STAT}} = 268.26 \) is greater than the critical bound 3.114, reject \( H_0 \).

There is enough evidence to conclude that there is a difference in the mean compressive strength after 2, 7 and 28 days.

(b) From Table E.10, \( Q_\alpha = 3.4 \).

\[
\text{critical range} = Q_\alpha \sqrt{\frac{\text{MSE}}{r}} = 3.4 \sqrt{\frac{0.0944}{40}} = 0.1651
\]

\[
|\overline{X}_1 - \overline{X}_2| = 0.5531^* \quad |\overline{X}_1 - \overline{X}_3| = 1.5685^* \quad |\overline{X}_2 - \overline{X}_3| = 1.0154^*
\]

At the 0.05 level of significance, all of the comparisons are significant. This is consistent with the results of the \( F \)-test indicating that there is significant difference in the mean compressive strength after 2, 7 and 28 days.

11.36 Excel Two-way ANOVA output:

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Source of Variation} & \text{SS} & \text{df} & \text{MS} & \text{F} & \text{P-value} \\
\hline
\text{Sample} & 52.5625 & 1 & 52.5625 & 23.57944 & 0.000394 \\
\text{Columns} & 1.5625 & 1 & 1.5625 & 0.700935 & 0.418832 \\
\text{Interaction} & 3.0625 & 1 & 3.0625 & 1.373832 & 0.2639 \\
\text{Within} & 26.75 & 12 & 2.229167 & & \\
\text{Total} & 83.9375 & 15 & & & \\
\hline
\end{array}
\]

(a) \( H_0: \) There is no interaction between development time and developer strength.

\( H_1: \) There is an interaction between development time and developer strength.

Decision rule: If \( F_{\text{STAT}} > 4.747 \), reject \( H_0 \). Test statistic: \( F_{\text{STAT}} = 1.374 \).

Decision: Since \( F_{\text{STAT}} = 1.374 \) is less than the critical bound of 4.747, do not reject \( H_0 \). There is insufficient evidence to conclude that there is any interaction between development time and developer strength.

(b) \( H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2 \)

Decision rule: If \( F_{\text{STAT}} > 4.747 \), reject \( H_0 \). Test statistic: \( F_{\text{STAT}} = 23.58 \).

Decision: Since \( F_{\text{STAT}} = 23.58 \) is greater than the critical bound of 4.747, reject \( H_0 \). There is sufficient evidence to conclude that developer strength affects the density of the photographic plate film.

(c) \( H_0: \mu_{10} = \mu_{14} \quad H_1: \mu_{10} \neq \mu_{14} \)

Decision rule: If \( F_{\text{STAT}} > 4.747 \), reject \( H_0 \). Test statistic: \( F_{\text{STAT}} = 0.701 \).
Decision: Since $F_{STAT} = 0.701$ is less than the critical bound of 4.747, do not reject $H_0$. There is inadequate evidence to conclude that development time affects the density of the photographic plate film.

(e) At 5% level of significance, developer strength has a positive effect on the density of the photographic plate film while the developer time does not have any impact on the density. There is no significant interaction between developer time and developer strength on the density.