AUTOMATIC CONTOURING OF TWO-DIMENSIONAL FINITE STRAIN DATA ON THE UNIT HYPERBOLOID AND THE USE OF HYPERBOLOIDAL STEREOGRAPHIC, EQUAL-AREA AND OTHER PROJECTIONS FOR STRAIN ANALYSIS

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Two-dimensional strain data are plotted on a cartesian $R\phi$ graph (Dunnett, 1969) or a polar strain graph (Elliott, 1970). A logarithmic scale is related to the deviatoric natural strain, $\epsilon$ (Nadai, 1950). Yamaji (2008) showed that a two-dimensional unit hyperboloid $H^2$ provides a unifying parameter space. Points on $H^2$ are $x = (x_0, x_1, x_2)^T$, with origin C. If strain is represented by $(\rho, \psi) = (\ln R, 2\phi)$, then an ellipse is $x = (\cosh \rho, \sinh \rho \cos \psi, \sinh \rho \sin \psi)^T$. Reynolds (1993) gave equidistant, equal-area, orthographic, gnomic and stereographic azimuthal projections to map $H^2$ to the $x_1x_2$ plane. The equidistant is the Elliott graph, it preserves radial distance so $\epsilon$ is undistorted. Wheeler (1984) discussed the orthographic. The equal-area distorts $\epsilon$ but preserves area for comparing densities. The stereographic is conformal. Curves of equal distance from C remain circles with strain, so for a symmetrical distribution the centroid of the projected data is the best-fit ellipse. Projecting $H^2$ onto a surface whose axis is parallel to $x_0$ gives a family of cartesian graphs. The equidistant is the $R\phi$ graph. The centroid in none of these graphs is a good estimator of the best-fit ellipse.

Elliott (1970) hand-contoured strain data on the polar graph to bring out indications of pre-strain fabrics. It is desirable to have a method that is rapid, reproducible, and based on the underlying geometry of the data, rather than the projection. $H^2$ provides a measure of distance directly related to strain, $d_H = \cosh^{-1} (-a \cdot b)$, analogous to a great-circle distance on a sphere. By analogy with methods for spherical orientation data (Diggle and Fisher, 1985; Fisher et al., 1987; Vollmer, 1995), contouring strain data can be done by back-projecting a grid onto $H^2$ using inverse functions. The distances from each node to each data point $x_k$ are summed to determine the node value, $f_{ij}$, using a weighting function, $w_k$, with parameter $\kappa$, based on the cumulative distribution function for $H^2$ (Jensen, 1981). To account for sample size, $n$, $\kappa$ is replaced with a normalized parameter: $\kappa_n = \kappa n^{1/3}$, by analogy with the spherical case (Fisher et al., 1987). The $f_{ij}$ values are contoured as percentages of the maximum $f_{ij}$ value. A computer program, EllipseFit, that implements these methods is freely available.