Managing Facilitating Goods

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Managing Facilitating Goods

Learning Objectives
- Describe the functions and costs of an inventory system.
- Determine the order quantity.
- Conduct an ABC analysis of inventory items.
- Determine the order quantity for the single-period inventory case.
- Describe the rationale behind the retail discounting model.

Role of Inventory in Services
- Decoupling inventories
- Seasonal inventories
- Speculative inventories
- Cyclical inventories
- In-transit inventories
- Safety stocks

Considerations in Inventory Systems
- Type of customer demand
- Planning time horizon
- Replenishment lead time
- Constraints and relevant costs

Relevant Inventory Costs
- Ordering costs
- Receiving and inspections costs
- Holding or carrying costs
- Shortage costs
Inventory Management Questions

- What should be the order quantity (Q)?
- When should an order be placed, called a reorder point (ROP)?
- How much safety stock (SS) should be maintained?

Inventory Models

- Economic Order Quantity (EOQ)
- Special Inventory Models
  - With Quantity Discounts
  - Planned Shortages
- Demand Uncertainty - Safety Stocks
- Inventory Control Systems
  - Continuous-Review (Q, r)
  - Periodic-Review (order-up-to)
- Single Period Inventory Model

Inventory Levels For EOQ Model

EOQ Formula

- Notation
  - \( D \) = demand in units per year
  - \( H \) = holding cost in dollars/unit/year
  - \( S \) = cost of placing an order in dollars
  - \( Q \) = order quantity in units
- Total Annual Cost for Purchase Lots
  \( TC_p = S \left( \frac{D}{Q} \right) + H \left( \frac{Q}{2} \right) \)
- EOQ
  \( EOQ = \sqrt{\frac{2DS}{H}} \)

Annual Costs For EOQ Model

Annual Costs for Quantity Discount Model
Inventory Levels For Planned Shortages Model

Formulas for Special Models

- **Quantity Discount Total Cost Model**
  \[ TC_{qd} = CD + S(D/Q) + I(CQ/2) \]

- **Model with Planned Shortages**
  \[ T_{C} = S \left( \frac{D}{Q} + H(Q-K)^2 + B \frac{K}{2Q} \right) \]
  \[ Q^* = \frac{2DS}{H(B+H)} \quad K^* = Q \left( \frac{H}{H+B} \right) \]

Values for Q* and K* as A Function of Backorder Cost

<table>
<thead>
<tr>
<th>S</th>
<th>Q*</th>
<th>K*</th>
<th>Inventory Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>B ↠ 0 undefined</td>
</tr>
<tr>
<td>∞</td>
<td>0</td>
<td>0</td>
<td>undefined</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Demand During Lead Time Example

Safety Stock (SS)

- **Demand During Lead Time (LT) has Normal Distribution**
  \[ \text{Mean}(d) = \mu(LT) \]
  \[ \text{Std. Dev.}(\sigma) = \sigma \sqrt{LT} \]

- **SS with r% service level**
  \[ SS = z \sigma \sqrt{LT} \]

- **Reorder Point**
  \[ ROP = SS + d_L \]

Continuous Review System (Q,r)
Inventory Control Systems

Continuous Review System

\[
EOQ = \sqrt{\frac{2DS}{H}}
\]

\[
ROP = SS + \mu LT
\]

Periodic Review System

\[
TIL = SS + \mu (RP + LT)
\]

\[
SS = z \sigma \sqrt{LT}
\]

Single Period Inventory Model

Newsvendor Problem Example

\[ D = \text{newspapers demanded} \]
\[ P(D) = \text{probability of demand} \]
\[ Q = \text{newspapers stocked} \]
\[ P = \text{selling price of newspaper, $10} \]
\[ C = \text{cost of newspaper, $4} \]
\[ S = \text{salvage value of newspaper, $2} \]
\[ Cu = \text{unit contribution: } P-C = $6 \]
\[ Co = \text{unit loss: } C-S = $2 \]

ABC Classification of Inventory Items

AB Classification of Inventory Items

<table>
<thead>
<tr>
<th>Inventory Items</th>
<th>Unit cost</th>
<th>Sales ($)</th>
<th>Dollar Volume ($)</th>
<th>Percent of Dollar Volume</th>
<th>Percent of SKUs</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computers</td>
<td>3000</td>
<td>50</td>
<td>150,000</td>
<td>74</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>Entertainment center</td>
<td>2500</td>
<td>30</td>
<td>75,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Television sets</td>
<td>400</td>
<td>60</td>
<td>24,000</td>
<td>12</td>
<td>10</td>
<td>B</td>
</tr>
<tr>
<td>Refrigerators</td>
<td>1000</td>
<td>15</td>
<td>15,000</td>
<td>6</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Monitors</td>
<td>200</td>
<td>50</td>
<td>10,000</td>
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<tr>
<td>Stereos</td>
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<td>9,000</td>
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<tr>
<td>Cameras</td>
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<td>Software</td>
<td>50</td>
<td>100</td>
<td>5,000</td>
<td>2</td>
<td>10</td>
<td>C</td>
</tr>
<tr>
<td>Computer disks</td>
<td>1006</td>
<td>5,990</td>
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</tr>
</tbody>
</table>

ABC Classification of Inventory Items

<table>
<thead>
<tr>
<th>Percentage of inventory items (SKUs)</th>
<th>Percentage of dollar volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
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<tr>
<td>20</td>
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<td>30</td>
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<td>40</td>
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<td>50</td>
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<td>60</td>
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<tr>
<td>70</td>
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<tr>
<td>80</td>
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<tr>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
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</tbody>
</table>

Single Period Inventory Model

Expected Value Analysis

<table>
<thead>
<tr>
<th>Order</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>130</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
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<tr>
<td>2</td>
<td>30</td>
<td>50</td>
<td>70</td>
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<tr>
<td>3</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
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<tr>
<td>4</td>
<td>10</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Profit</th>
<th>$21.54</th>
<th>$34.43</th>
<th>$35.77</th>
<th>$35.99</th>
<th>$36.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>$21.54</td>
<td>$34.43</td>
<td>$35.77</td>
<td>$35.99</td>
<td>$36.33</td>
<td></td>
</tr>
</tbody>
</table>
Single Period Inventory Model

Incremental Analysis

- \( E \) (revenue on last sale) \( \geq \) \( E \) (loss on last sale)
- \( P \) (revenue) (unit revenue) \( \geq \) \( P \) (loss) (unit loss)

\[
\begin{align*}
P(D \leq Q) & \geq P(D \geq Q) \\
P(D \leq Q) & \geq P(D \leq Q) \\
1 - P(D < Q) & \geq P(D < Q)
\end{align*}
\]

(Critical Fractile)

- \( C_u \) = unit contribution from newspaper sale (opportunity cost of underestimating demand)
- \( C_o \) = unit loss from not selling newspaper (cost of overestimating demand)
- \( D \) = demand
- \( Q \) = newspaper stocked

Critical fractile for the Newsvendor Problem

Retail Discounting Model

- \( S \) = current selling price
- \( D \) = discount price
- \( P \) = profit margin on cost (% markup as decimal)
- \( Y \) = average number of years to sell entire stock of “dogs” at current price (total years to clear stock divided by 2)
- \( N \) = inventory turns (number of times stock turns in one year)

\[
\text{Loss per item} = \text{Gain from revenue} \\
S - D = D(PNY)
\]

\[
D = \frac{S}{(1 + PNY)}
\]