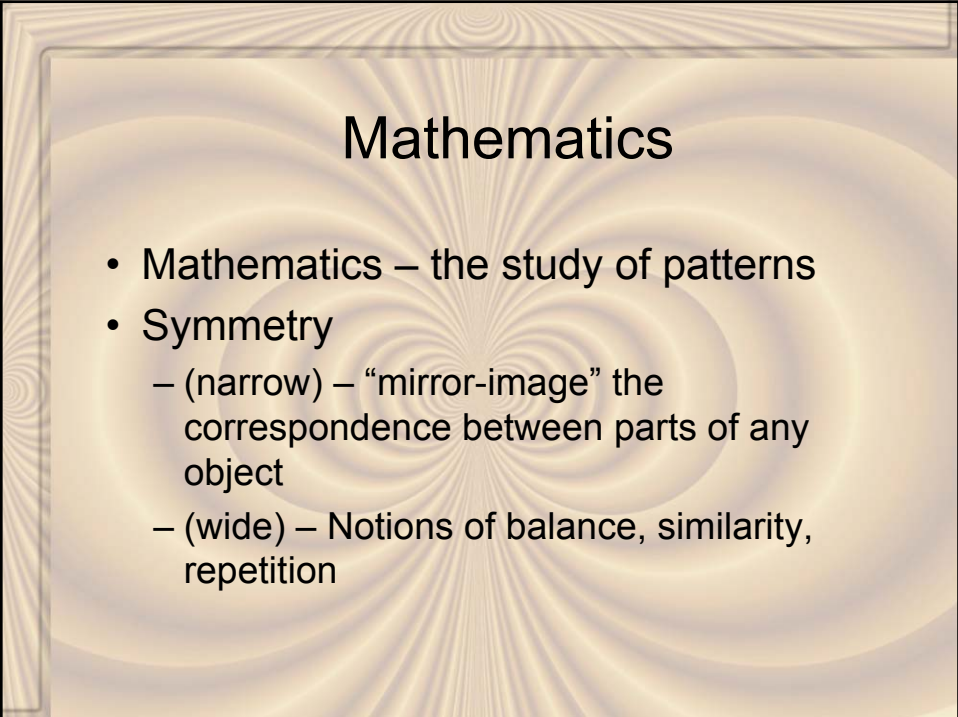




# Symmetry and Patterns

## Chapter 19



## Mathematics

- Mathematics – the study of patterns
- Symmetry
  - (narrow) – “mirror-image” the correspondence between parts of any object
  - (wide) – Notions of balance, similarity, repetition

## Spirals

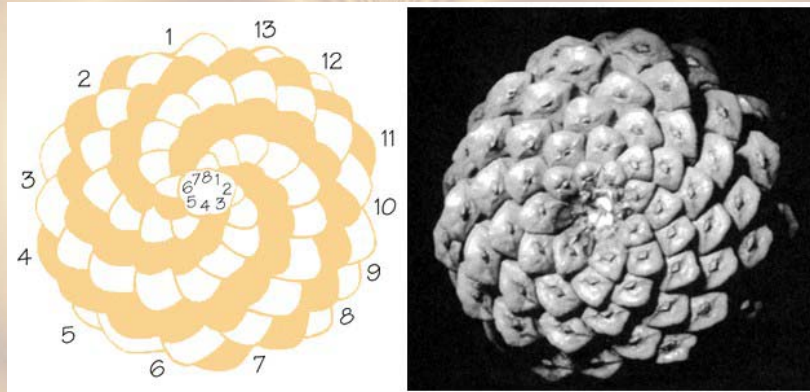


- Spiral pattern – same shape at *any* size

## Spirals

- Phyllotaxis – spirals are geometrically similar to one another and arranged in a regular way with balance and “proportion”.
- Phyllotaxis demonstrates rotational symmetry.

## Phyllotaxis (cont)



## Fibonacci



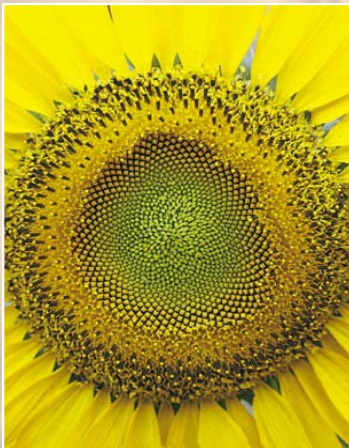
- Leonardo Pisano Bigollo
  - Greatest Mathematician of the Middle Ages
  - Introduces Arabic numbers to Europe
  - Descent from ancestor “bonnacci”

## Fibonacci Sequence

- Start with the sequence 1, 1
- Then add previous 2 numbers to create the third
- Sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

$$F_1 = 1, F_2 = 1, F_{n+1} = F_n + F_{n-1} \text{ for } n \geq 2$$

## Fibonacci



- Sunflower
  - 55 one direction
  - 89 in the other direction
- Pineapple
  - 8 right
  - 13 left
  - 21 vertically

## Balance in Symmetry

- The Sunflower spiral shows
  1. Similarity
  2. Repetition
  3. Balance – regularity in *how* the repetitions are arranged

## The Golden Ratio

(Divine Proportion)

- Greeks – found a balance in Ratios
- Golden Ratio – proportion essential to beauty and symmetry
- Golden Ratio – becomes the standard for beautiful proportions

## PHI(fi)

- Phi  $\phi = 1 + \frac{\sqrt{5}}{2} = 1.618$



Phi

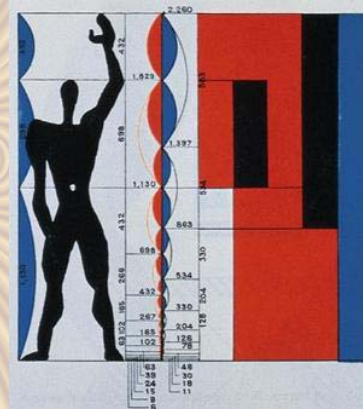
## Balance in Ratios

- Arithmetic Mean – add all values and divide by the number of values
- Greeks preferred the Geometric Mean
- S= side of a square with the same area as a rectangle of L\*W

$$LW = S^2$$

# Human Body

- Body – ratio close to Golden Ratio
- Total height = Golden Ratio
- Height to navel = 1



# Rigid Motion

- Rigid Motions (isometry = “same size”) Specific kind of variation on the original pattern (e.g. move it, rotate it, flip it, etc) **BUT** don’t change its size or shape.
- What can we do to a pattern without changing its appearance?

## Rigid Motion

- Rigid Motion – one that preserves the size and shape of the figures (patterns)
  - Reflection (across a line)
  - Rotation (around a point)
  - Translation (in a particular direction)
  - Glide Reflection (across a line)

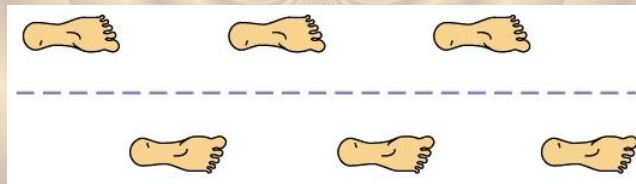
## Rigid motions

- All reflections and all rotations are rigid motions
- Translations are rigid – moving every point in the plane a certain distance in the same direction – known as glide



## Glide Reflection

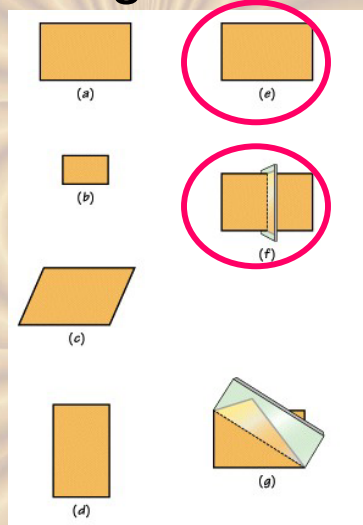
- Glide reflection – translation (glide) with reflection across a line that is parallel to the direction of the translation



## Preserving the Pattern

- Preserve the pattern – the pattern must look exactly the same after the rigid motion. All of the parts must appear in the same place

## Preserving the Pattern?



## Patterns with repetition

- Motif – individual element or figure of a design
- Pattern (of the design) how the copies of the Motif are arranged

## Analyzing Patterns

- Patterns in the plane can be divided into those that have indefinitely many repetitions in
  1. No direction – rosette patterns
  2. Exactly one direction – strip patterns
  3. More than one direction – wallpaper patterns

## Rosette pattern

- Flowers
  - reflection
  - motif =  $\frac{1}{2}$  petal



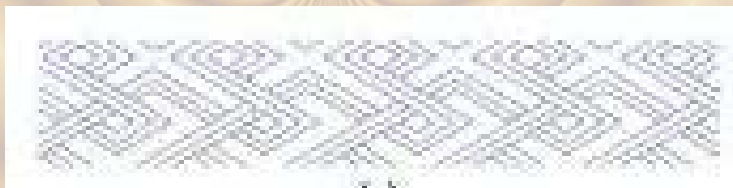
## Rosette pattern

- Pinwheel
  - no reflection
  - motif – entire petal



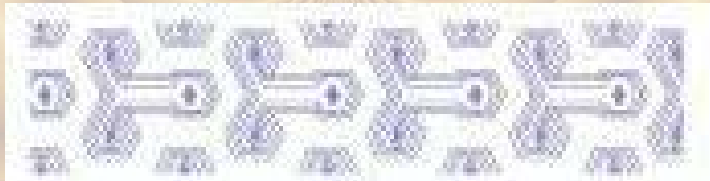
## Strip Patterns

- All strip patterns have
  - Repetition symmetry
  - Translation symmetry along the direction of the strip



## Strip Patterns

- All strip patterns have
  - Repetition symmetry
  - Translation symmetry along the direction of the strip
- Add
  - Reflection – on the center horizontal line



## Strip Pattern Combinations

- Vertical Line Reflection
- Rotational Symmetry (half turn)
- Glide reflection
- **NO** horizontal line reflection



## Symmetry Groups

- Symmetries of a pattern have the following properties
  1. If we combine two symmetries we get another symmetry
  2. There is an Identity (null) Symmetry – doesn't change anything

## Symmetry Groups (cont)

- Symmetries of a pattern have the following properties
  3. Each symmetry has an inverse (opposite) that undoes it and preserves the pattern
    - e.g. reflection by its own inverse
    - glide undone by another glide of the same distance in the opposite direction
  4. We may combine consecutive symmetries without affecting the result

# Symmetries

- Repetition symmetry
- Translation symmetry
- Reflection – on the center horizontal line
- Reflection – on the vertical axis
- Rotational Symmetry –  $180^\circ$
- Glide Reflection

# Group of numbers (Symmetry Groups)

- Numbers have the following properties  
(Symmetries of a pattern have the following properties)
  1. Multiply two positive real numbers gives another positive real number (If we combine two symmetries we get another symmetry)
  2. The positive real number 1 is an identity element. There is an Identity (null) Symmetry – doesn't change anything

# Group of numbers

## (Symmetry Groups)

- Numbers have the following properties (Symmetries of a pattern have the following properties)
3. Any Positive real number  $x$  has an inverse  $1/x$  in the collection. (Each symmetry has an inverse (opposite) that undoes it and preserves the pattern)
  4. You may multiply several numbers together in any order. (We may combine consecutive symmetries without affecting the result)

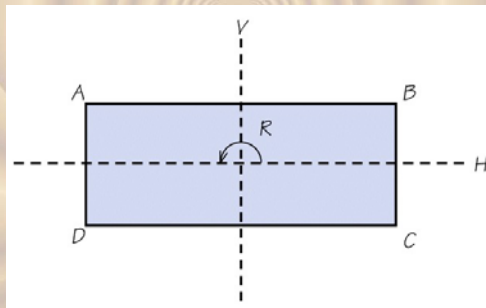
## Symmetry Group of a Rectangle

- The symmetries (rigid motions) that bring the rectangle back to coincide with itself (even as they interchange the labeled corners)
1. Identity symmetry  $I$  – leave all points where they are
  2. Half-turn rotation  $R$  around its center
  3. A reflection  $V$  in the vertical line through the center
  4. A reflection  $H$  in the horizontal line through the center



## Rectangle Group

- Combine any 2 symmetries you get a third
- Apply  $V$  and then  $H$  will get  $R$



## Rectangle Group

- Each element is its own inverse
- Apply  $V$  and then  $H$  and then  $R$
- Same as  $R$  and then  $H$  and then  $V$

