

Taking the PROBLEM Out of Problem Solving



A New Approach to
Problem Solving in the
Middle School Math
Class

Written by
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Taking the Problem out of Problem Solving: A New Approach to Problem Solving in the Middle School math Class.

SUNY New Paltz, 2001.

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Introduction

This booklet was written as part of an in-service project for the New York State K-8 Math Mentor Network. It is based on a method that I have been using in my classroom for the last two years. I began experimenting and developing this after seeing the sample exam for the New York State Math Assessment in 1998.

I realized that students needed better preparation for the part 2 and 3 questions. I did not find appropriate questions in our textbook and there were no review type books available, so I began experimenting with questions from the sample book. Almost immediately it became apparent that my **eighth** grade students were lacking in two critical areas: interpreting these questions and supplying written explanations for their work.

Over the summer of 1998 I began thinking of ways to improve these two skills. The students were doing journal writing in their Language Arts class and I decided to address their math skills by journal writing in the mathematics class. The students each purchased a composition notebook and, when time allowed, I gave them questions to work on. Initially the results were dreadful and I realized that I had to do more. It also became apparent that I did not enjoy carrying 120 journals home to correct.

During the following summer I determined that I needed to rethink my methods. First and foremost I realized that my students needed to be given more problems to work on, but I did not want to take time away from my teaching. I decided to give the students monthly assignments. These were set up with four or five questions per month, and collected at the end of the month. Secondly, I knew that I wanted to rid myself of the cumbersome journals so I instructed the students to do their work on loose-leaf paper. This was more convenient for me and made the chore of correcting easier.

This past year I changed the journal assignments from monthly to weekly. This has allowed me extra time to adequately present a variety of question types. I regularly give the assignments out on Friday with 4 or 5 questions that are due the following Friday. These questions may be all of one type, all on one theme, or simply a mixture of questions. I have used some questions that I found in review books that are now available and past tests. However, the majority of questions are ones that I make up based on what we are currently doing in class.

The scores of my students from the 1999 test to the 2000 test rose dramatically. We had a 28% increase in the number of students reaching scores of 3 and 4. This year the journal averages of my students rose 11.5 points between the first and third quarters. I am hoping this will lead to further improvement in the scores on the New York State Assessment.

Problem Solving in the Middle School

1. LEARNING CONTEXT:

Key Idea 1 - Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students will use a variety of reasoning strategies including:

- a) Use pictures, diagrams, or charts.
- b) Look for patterns.
- c) Use guess and check.
- d) Work backwards.
- e) Use logical reasoning.
- f) Use formulas.
- g) Write and solve equations.

Students will support their solutions by:

- a) Written explanations.
- b) Showing work.
- c) Checking solutions.

Through the use of weekly journal assignments students will prepare for parts 2 and 3 of the New York State Grade 8 Mathematics Assessment. Problems will be chosen based on current classroom topics and previous knowledge. These problems can be teacher designed, textbook questions, previous NYS Assessment examples, or real-life situations. Timely topics often make the questions more relevant to the students and provide a means of showing why mathematics is relevant to their life. Choosing topics that relate to other subject areas demonstrates the cross-curriculum importance of mathematical knowledge and reasoning. Science and technology are often easy to incorporate into the journal questions, but it is important that other subject areas are used as often as possible.

Prior to assigning the first journal work students need to be introduced to a variety of problem solving techniques. Other methods can be taught throughout the year. Students should also be given a format to use when solving problems (See Appendix A).

Each problem will be graded using either the 2- or 3-point rubric provided by New York State (See Appendix B). These should be distributed to the students prior to the first assignment and explained in detail. For use in determining report card grades these can be converted into a percent at the end of the marking period and counted as 25% of their grade.

2. PROCEDURE

Journal assignments will be distributed on Fridays. Five questions per week will be assigned and collected on the following Friday. Journals handed in before Wednesday can be corrected by the teacher and returned to the students, allowing them to rework those problems they have not correctly solved or completed. Journals will be returned to students on Mondays and reviewed in the class. Grades will be recorded and used to determine marking period grades.

3. INSTRUCTIONAL/ENVIRONMENTAL MODIFICATIONS

No structural modifications will be made in the development of journal assignments. Student modifications, as provided by Individual Educational Plans, will be used as they relate to student work and the New York State Assessment.

4. MATERIALS AND SUPPLIES

For ease of correcting journals, students should do their work on loose-leaf paper rather than in notebooks. After classroom review, these should be placed in individual folders and kept by the teacher. A table of contents should be kept in the front of each folder, where students should record the date of each type of question (See Appendix C). Periodically, students should be asked to review their work and look at examples of certain types of problems. They can rework these problems, checking for improvement. At least once a marking period teachers should review these journals, checking for improvement.

6. TIME REQUIRED

Before the initial journal assignment teachers should spend time discussing the scoring rubrics and problem-solving format. This will take approximately one day. Review or introduction of types of questions should follow. A week should be allowed for this with appropriate examples assigned for both class work and homework.

Each week approximately 10 minutes should be set-aside on Fridays so that questions can be read with the class, to insure that students understand the problems. When the journals are returned on Mondays teachers need to set aside enough time to thoroughly explain each solution and answer questions that the students may have.

The weekly assessment of the journal assignments will require between 5 and 10 minutes per student. This allows the teacher to not only check for accuracy but also to make appropriate comments. Although spelling, grammar; punctuation, and other rules of English need not be evaluated, these should be noted. Teachers may choose to use a rubric designed with the Language Arts teacher to include an evaluation of Language Arts skills (See Appendix D).

7. REFLECTION

I have found that the use of journals in math allows me to continuously work on problem

solving skills. It encourages the introduction of new methods while reviewing previously learned methods. This constant exposure to problems that use a variety of reasoning strategies allows the students to become more accomplished and more secure in their problem solving methods. Students become used to the weekly assignment and the fear that I used to see in students when I talked about “word problems” has diminished greatly.

As new topics are introduced into the classroom a larger variety of problems can be used. All of the Key Ideas are eventually addressed in the journals by the inclusion of new types of questions. This leads to a better understanding of the importance and relevance of new material, and, hopefully, results in higher scores on the New York State Eighth Grade Mathematics Assessment.

Appendix A

Problem Solving Format

What you know.	Diagram, chart, pictures, sequence of numbers, formulas, equations.
What you are asked to find?	Work Solution
Explanation or Check	

Appendix B

Holistic Rubrics

2-Point Rubric

- 2- Points A two point response is complete and correct. This response
- demonstrates a thorough understanding of the mathematical concepts and/or procedures embodied in the task
 - indicates that the student has completed the task correctly, using mathematically sound procedures
 - contains clear, complete explanations and/or adequate work when
- 1-Point A one point response is only partially correct. This response
- indicates that the student has demonstrated only a partial understanding of the mathematical concepts and/or procedures embodied in the task
 - addresses some elements of the task correctly but may be incomplete or contain some procedural or conceptual flaws
 - may contain an incorrect solution but applies a mathematically appropriate process
 - may contain a correct numerical answer but required work is not provided
- 0-Points A zero point response is completely incorrect, irrelevant, or incoherent, or a correct response arrived at using an obviously incorrect procedure.

3-Point Rubric

3-Points A three point response is complete and correct. This response

- demonstrates a thorough understanding of the mathematical concepts and/or procedures embodied in the task
- indicates that the student has completed the task correctly, using mathematically sound procedures
- contains clear, complete explanations and/or adequate work when required — _____

2-Points A two-point response is partially correct.

This response

- demonstrates partial understanding of the mathematical concepts and/or procedures embodied in the task
- addresses most aspects of the task, using mathematically sound procedures
- may contain an incorrect solution but applies a mathematically appropriate process with valid reasoning and/or explanation
- may contain a correct solution but provides incomplete procedures reasoning, and/or explanations
- may reflect some misunderstanding of the underlying mathematical concepts and/or procedures

1-Point A one-point response is incomplete and exhibits many flaws but is not completely incorrect.

This response

- demonstrates only a limited understanding of the mathematical concepts and/or procedures embodied in the task
- may address some elements of the task correctly but reaches an inadequate solution and/or provides reasoning that is faulty or incomplete
- exhibits multiple flaws related to a misunderstanding of important aspects of the task, misuse of mathematical procedures or faulty mathematical reasoning
- reflects a lack of essential understanding of the underlying mathematical concepts
- may contain a correct numerical answer but required work is not 0-Points provided

0-Points A zero-point response is completely incorrect, irrelevant or incoherent, or a correct response that was arrived at using an obviously incorrect procedure.

Appendix D

Sample Journal,

Correct Answers.

Journal

Due Feb. 16,2001



1) Abraham Lincoln was born on Feb. 12, 1809. He was elected as the 16th president and took the oath of office on March 4, 1861. John Wilkes Booth, a well-known actor, shot Lincoln on April 14, 1865, and President Lincoln died the following day.

- a) How old was he when he died?
- b) How many days did he serve as president?

2) When Abraham Lincoln died vice president Andrew Johnson became President. He was a very controversial person, and angered many Northerners during Reconstruction. As a result of some of his decisions he became the first president to be impeached. During an impeachment trial the senate must convict the accused by a $\frac{2}{3}$ majority of the senate. Johnson was found not guilty by 1 vote. Recently, former President Bill Clinton was also impeached, and also found not guilty.

- a) How many senators are there now?
- b) What is the least number of senators that could have voted not guilty for President Clinton?

3) Valentine's Day is celebrated on February 14th. Many people celebrate this by sending cards, flowers, or candy. Last year I sent my daughter, Barby, a dozen red roses. The cost for this was \$38.50. I got my mother 5 yellow roses (one for each of her children) and these cost \$17.25. Which of these was a better deal?

4) Vacation starts this Friday. Many people go to a warmer climate for February vacation. A friend of mine is going to drive to Florida and then take a 5 day cruise.

- a) Use a map and approximate the distance from Albany to Miami.
- b) If my friend drives at an average speed of 60 miles per hour, how many hours will it take him to get to Miami?
- c) Explain why the time you got for b) is probably not long enough. Give at least 2 reasons.

5) During vacation I plan to read as many books as possible. I can read about 50 pages an hour. The average book has about 300 pages.

- a) About how long will it take me to read a book?
- b) I have 5 long books (averaging 400 pages each) and 7 shorter books (averaging 225 pages) that I want to read. Approximately how many days will this take me, if I read about 3 hours a day?

ANSWERS:

1a) 56 years old when he dies

1b) president for years, of which one was a leap year, resulting in $(4 \times 365 + 1 =)$ 1461 days and in addition from 3/4/65 to 4/15/65 (42 days) makes for a total of $1461 + 42 = 1503$ days.

2a) 50×2 per state = 100

2b) a little more than $1/3$ for not guilty, $1/3$ of 100 = $33 \frac{1}{3}$ senators, which is at least 34 senators.

3. $38.5/12 = 3.21$ and $17.25/5 = 3.45$. A dozen at \$38.50 is a better deal per rose.

4a) acceptable answers are between 1200 and 1600 miles

4b) take acceptable answer from part 4a) and divide by 60.

4c) Students cannot use changing speeds as one of the two reasons, since the 60mph was given as an average.

5a) $300/50 = 6$ hours

5b) $(400 \times 5) + (225 \times 7) = 2000 + 1575 = 3575$ pages

$$3575/50 = 71.5 \text{ hours}$$

$$71.5/3 = 23.83 \text{ or } 24 \text{ days.}$$

Appendix E

Sample Questions

- 1) General Questions
- 2) Questions by Key Idea
- 3) Thematic Questions
- 4) Cross Curricular Questions

General Questions

Each set of questions that follows can be given on a monthly basis or as Problem Of the Week (POW), or intensified as daily problems. As much as possible use the general rubrics included in this document for rating your students' work.

SET 1.

1. A new middle school is being built in a town for about 1,500 students. The students decide to take a survey to determine the school mascot. They plan to go to the mall on Tuesday at noon. They will ask the first 20 people they see what the school mascot should be. This is ***not*** a good way to determine the school mascot. Explain in paragraph form what they are doing wrong and what they should do to do a better survey.
2. There is a mistake in the solution of the inequality shown below. Carefully copy the problem into your journal. Circle the first incorrect line. Then do the problem over, correcting the mistake, and finish solving the problem.

$$\begin{aligned}7 - 3x - 3 &< 20 - 4 \\4 - 3x &< 16 \\3x &< 12 \\x &< 4\end{aligned}$$

3. The table shown below shows the amount of the discount given for every CD Danielle buys at the store. Each CD costs \$14.00.

# of CD's bought	Discount in \$
2	\$3
3	\$5
4	\$7
5	\$9

- a) What is the rule for finding the discount?
 - b) Explain how you determined this rule.
4. Suppose that your scores in math class are 40, 90, 91, 89, and 92.
 - a) What is your mean score?
 - b) What is your median score?
 - c) Write a brief note to your teacher explaining why your median score is a better indicator of your ability than your mean score.

SET 2

1. Three coins are tossed. Two of the coins are “fhir” with heads (H) on one side and tails (T) on the other. The third coin has heads (H) on both sides.
 - a) Draw a tree diagram showing the possible outcomes when the three coins are flipped at the same time.
 - b) Which of the following outcomes is ***most*** likely to occur?
 - 3 heads
 - 2 heads and 1 tail
 - 1 head and 2 tails
 - 3 tails
 - c) Which of the outcomes listed in part b) is ***least*** likely to occur?
2. A newspaper reports that in 1997 half of the families in the USA had an income of \$37,006 or more.
 - a) Is \$37,006 the ***mean, median*** or ***mode*** of the family incomes? Explain how you know this?
 - b) Someone says, “Since \$0 is the lowest possible income, then twice \$37,006 or \$74,012 must be the highest income.” Is this statement true or false? Explain how you know this.
3. As autumn comes, the number of ducks heading south increases 3 times for every 5° F drop in the temperature. If 2,100 ducks head south when it is 35° F, ***about*** how many ducks will head south when the temperature is 25° F? Show your work **or** explain how you got your answer.
4. Nicholas is looking over his training schedule for the Ironman Triathlon. Every Monday, Wednesday and Friday he runs for 120 minutes. Every Tuesday and Thursday he does 3 sets of 60 minutes of swimming. Monday through Friday he rides his bike for 45 minutes three times a day. How many ***hours*** per week does he train?

SET 3

1. Each year, approximately 8 million plastic bottles are thrown away in the state of California. On average, about how many plastic bottles are thrown away each hour in California? Round the answer to the nearest whole number.
2. Terry collects baseball cards. The number of cards he collected during each of the last four months is shown below.

<u>MONTH</u>	<u>NUMBER OF CARDS</u>
March	17
April	20
May	16
June	18

Estimate the number of cards Terry will have collected by the end of this year (through December).

3. Brent wants to buy a stereo system that costs \$499. So far, he has saved \$75. Brent earns \$35 each week working six hours on Saturday.
 - a) How many weeks will it take Brent to save up enough to buy the stereo?
 - b) After saving for 6 weeks, Brent's father agrees to loan Brent the extra money he needs to buy the stereo. How much will Brent need to borrow?
4. Leeanne bought 7 boxes of chocolates. She bought small boxes for \$3.50 each and large boxes for \$5.00 each. She paid a total of \$27.50. How many of each size box did Leeanne buy?

SET 4

1. John has a large jar of cookies. He and 2 friends eat 4 cookies each. Then John bakes another batch of 15 cookies and puts them in the jar. This brings the total of cookies in the jar to 17. How many cookies were *originally* in the jar?

2. Study the pattern below. What are the next 3 numbers? Explain in words how you did this.

2, 6, 10, 14, 18, 22, __, __, __

3. A bicycle shop charges \$20.00 per hour for repair work. In addition, they charge a \$7.00 flat fee to determine what repairs are needed before they begin work. Complete the following chart for each of the first five hours of repair work. Let x equal the number of hours worked and let y equal the total cost.

X =	1	2	3	4	5
Y =					

4. I am a 2-digit number less than 50. The sum of my digits is 10. The product of my digits is 24. What number am I?

SET 5

1. Given the problem below, which of these numbers is the value represented by the ? Explain how you determined your answer.

$$-2 = 3$$

$$4 = 18$$

- A) 7 B) 11 C) 12 D) 18

2. Study the pattern below. What are the next 3 numbers? Explain how you determined your answer.

1, 3, 9, 27, __, __, __

3. After working one summer, two students open their own savings accounts. The table below shows the amount of money each student had in their account:

Student's Name	Original Savings	After 1 Month	After 2 Months	After 3 Months	After 4 Months
Joe	100	106	112	118	124
Sam	100	96	92	88	84

If this pattern continues, what will the combined total of their savings be after 12 months? Show all work.

4. Noah is looking over his exercise routine. Every Monday, Wednesday, and Friday he jogs for 30 minutes. Every Tuesday and Thursday he does 2 sets of 15 minutes of rowing. Monday through Friday he does 3 sets of 5-minute stretches. How many hours per week does he train? Show all work.
5. Gwen has 20 meters of fencing. She wants to enclose the greatest possible area within the fence. What is the greatest area for a rectangle made from 20 meters of fencing?

SET 6

1. If there are 10 hotdogs in each package and 8 buns in each package, what is the least number of packages of each you can buy so that you have the same number of hotdogs and buns?
2. For each purchase, The Online Store gives customers bonus points that can be used toward a later purchase. Look at the table below:

Gift Value	Points Needed
\$1	50
\$2	90
\$3	120
\$4	160
\$5	190
\$6	230

If the pattern continues, and Jill has 400 points, what gift value has she achieved? Explain how you get your answer.

3. Blocks are piled up in the following way:

If this pattern continues, how many blocks will be in the eighth pile? Show your work.

4. Every time a baby is born it has a chance of being a boy and chance of being a girl. A family has 5 children, all girls, and is expecting another one soon. What is the chance that this baby will be another girl? Explain how/why you determined your answer.
5. A delivery truck leaves a store and travels in the following directions: 6 miles west, 8 miles south, 10 miles east, and then 8 miles north. At the end of the route, how far is the truck from the store? Show or explain how you got your answer.

SET 7

1. Shauna and Stacey are looking at posters and talking about their poster collections. Shauna says, "If I buy this poster and 2 more, I will have a total of 12." Stacey says, "If I buy this poster and 4 more, I will have a total of 17." If they don't buy any new posters, what is the total number of posters they have? Show your work.
2. Monica is saving to buy a bicycle. The bike costs \$500. She has \$214. Every week she receives \$7 allowance and \$20 for delivering papers. If she saves all the money she receives, how many more weeks will it take Monica to earn enough money to buy the bike? Show your work.
3. During the first 6 months it was open, Momma Mia's sold about 6,500 pizzas. Momma Mia's is open 7 days every week and sells about the same number of pizzas each day. About how many pizzas does Momma Mia's sell each day? Explain in words how you got your answer.
4. Larry wants his family to have lunch with him at school. Lunch prices for family members are shown below:

1 Guest	2 Guests	3 Guests	4 Guests	5 Guests
\$2.50	\$4.25	\$6.00		

If the guest price pattern continues, how much will Larry be charged to bring 5 family members to school for lunch? Describe in words the pattern you used.

5. The art club is selling paintbrushes. Small brushes sell for \$2.50 and large brushes sell for \$4.00. On Saturday, the art club sold 8 brushes and took in a total of \$24.50. How many of each type of brush did they sell on Saturday? Show your work.
6. The sum of two numbers is 38. The difference between the two numbers is 16. What are the two numbers?
7. Bacteria are single celled organisms. They reproduce by dividing in two. They do this every 20 minutes. Starting with one bacterium, how many would there be after 2 hours of division? (Hint: Make a table)
8. Melissa is decorating a hexagonal (6-sided) tablecloth. She will put 2 blue daisies at each

corner. She will put 5 white daisies along each edge. How many daisies will she use in all?

9. Terry collects baseball cards. The number of cards he collected during each of the last four months is shown below.

MONTH	NUMBER OF CARDS
March	17
April	20
May	16
June	18

Estimate the number of cards Terry will have collected by the end of this year (through December). Explain in paragraph form (at least 3 sentences are needed in a paragraph) how you did this.

10. Each year, approximately 8 million plastic bottles are thrown away in the state of California. On average, about how many plastic bottles are thrown away each hour in California? Round the answer to the nearest whole number. Show all your work. (Hint: A year has 365 days and a day has how many hours?)

SET 8

1. A school report says that in the last month the average number of days a student was absent was 1.8, but that most students were absent one or less days.

Part A: According to this report, what is the mean and median number of days that students were absent?

Mean _____ Median _____

Part B: Do the statements “the average number of days a student was absent was 1.8” and “most students were absent one or less days” contradict each other? Explain your answer.

2. If 1 inch equals 2.54 centimeters, how many feet are in 4 meters? Show all work.
3. In seven years, Charles will be half as old as his cousin. If his cousin is 27 years old now, how old is Charles now? Show all work.
4. December 6: Marie’s Record Shop is going out of business and must sell their entire record inventory. The chart below shows the number of records left at the end of each day of the sale. Copy the chart over onto your paper and continue it to determine on which day there will be only 27 records left in the shop.

Day	1	2	3	4
# of Records	19,683	6,561	2,187	729

5. Find the absolute value of the following expression. Show all work necessary to do this.

$$[(7)(-3) + (-9) - (-10)] + (-2)(8) - 16 + 4$$

SET 9

1. A photographer is arranging the eighth grade students for a class photograph. He put 10 people in the first row. He put 5 more people in the second row than in the first row. He continued adding 5 more people in each row until he had 8 rows in all. He put the 12 tallest students in the last row.

- What information in the problem is irrelevant?
- How many students are in the eighth grade photograph?
- Explain how you found your answer to part b.

2. Evaluate the following expression for $n = 3$ and $y = 2$. Show your work.

$$(24 \div 4 - y) \cdot n$$

Explain the steps you used to evaluate the expression.

3. The following chart shows the approximate masses of planets in our solar system. Order the planets from least mass to greatest mass. Show your work.

Mercury	3.30×10^{23}
Venus	4.87×10^{24}
Earth	5.98×10^{24}
Mars	6.42×10^{23}
Jupiter	1.90×10^{27}
Saturn	5.69×10^{26}
Uranus	8.69×10^{25}
Neptune	1.02×10^{26}
Pluto	1.32×10^{22}

SET 10

All of the following problems can be solved by looking for a pattern. For each problem, find the answer to the question, showing work when possible. Then Explain in words what the pattern is that let you find the answer.

1. Find the next 3 numbers in the following pattern: 1, 4, 9, 16, . . .
2. For a banquet a caterer pushes small tables together to make long tables. Each small table seats 4 people. The room can fit a long table made of 19 small tables pushed together. How many people can sit at this long table?
3. At the banquet, the caterer allows 3 desserts for every 2 people. Finish the table below to find how many desserts the caterer must make for 24 people.

People	2	4	6	8	10	12	14	16	18	20	22	24
Desserts	3	6	9	12								

4. Bus fare in a city is \$.30 for every 6 miles you travel. What is the fare for 36 miles?
5. Each week, a swim team increases the daily distance that each member swims. The first week, members each swim 500 meters a day. The second week, they swim 600 meters. The third week, they each swim 725 meters, and the fourth week, they swim 875 meters. If the pattern continues, how many meters per day will they swim in week 8?

Questions by Key Idea

Mathematical Reasoning

1. Diana wrote the following facts about three different numbers: a , b , and c .

Fact 1: $a \div c = a$
Fact 2: $a + b = a$
Fact 3: $abc = 1$

- A. If Fact 1 and Fact 2 are true, explain why Fact 3 is NOT true.
B. What are the values of each of the variables in Fact 1 and Fact 2?

2. Tony and his friends bought some snacks at a movie theater. The table below shows the snacks and their prices.

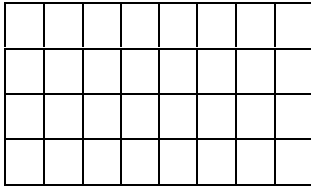
Item	Price
Popcorn	\$2.25
Candy bar	\$1.20
Soda	\$1.49
Cotton Candy	\$1.39
Nachos	\$3.69
Hotdog	\$2.07
Lemonade	\$0.99

Which items did Tony and his friends buy if together they spent exactly \$8.00 and bought only one of each item? (Show your work).

- 3a. List all the whole numbers from 1 to 100 that can be evenly divided by both 6 and 15.
3b. Describe at least one pattern that you notice in the set of numbers.
3c. How many whole numbers between 290 and 410 can be evenly divided by both 6 and 15? (Show your work).

Numbers and Numeration

1. Steve's dad is researching an old family recipe. He has found that members of the family do not agree on the amount of flour needed for the recipe. The following is a list of measurements he received from different people for the number of cups of flour needed: $1\frac{1}{4}$, $1\frac{1}{3}$, 1.5, $\frac{1}{3}$, $1\frac{1}{3}$. He wants to use the largest amount of flour. Which measurement above should he use? Explain how you determined which number was largest.
2. Shade in $\frac{5}{8}$ of the rectangle below:



What percent of the rectangle is shaded? Show your work.

3. Bryan and Annie are going to a school dance. Tickets cost \$12.00 per couple at the door, but are only \$10.50 if they are bought in advance. What percent of the door price will Bryan and Annie save if they buy their ticket in advance? Show your work.
4. In the equations below each of the variables a, b, c, and d represent a different value.

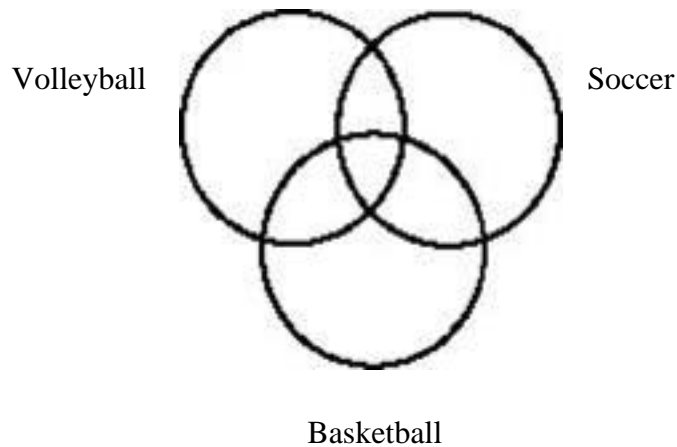
$a + c = b$
$b - a = c$
$c \times d = -d$

If $a = 4$, use the equations above to determine the value of each other variable. Show all work.

5. The physical education department distributed a survey asking the students in which three sports they participate: volleyball, soccer, or basketball. They received the following results:

- 29 students participate only in volleyball
- 31 students participate only in soccer
- 5 students participate in all three sports
- 2 students participate in soccer and basketball, but NOT in volleyball
- 1 student participates in volleyball and soccer, but NOT in basketball
- 39 students participate in volleyball
- 102 students participate in at least one of the three sports

Copy onto your paper and complete the Venn diagram to reflect the results of the survey.
Explain how to find the total number of students that play basketball.



Operations

1. Raoul and Clarence are writing questions for the “Name That Number” game. They want to ask the players to find a number with the following properties:

- The number is a one-digit positive integer.
- When 82 is divided by the number, the remainder is 3.

Does such a number exist? If so, write the number and explain your answer. If not, explain in words why there is no such number.

2. Georgina is collecting data on the growth of E. coli bacteria. Each E. coli cell reproduces by dividing in half every 20 minutes.

A. If Georgina starts with one E. coli cell, how many cells will be in the sample after 120 minutes?

B. After how many minutes will there be 256 cells in the sample?

3. In the expression below, insert two sets of parentheses so that the expression is equal to 15.

$$\frac{3 \times 18 + 3^2 - 6}{2 - 1 \times 5}$$

Now show the steps you used to simplify the expression so that it is equal to 15.

4. Each year approximately 28 million tons of yard waste (mowed grass, dead leaves, etc.) are thrown away in the United States. This is about 20% of the total amount of garbage thrown away in the United States each year. About how many tons of garbage are thrown away in the United States each year?

Modeling/Multiple Representation

- A) On graph paper, plot and label the following points.
A(-3, -2) B(7, -2) C(5,4) D(-5,4)

B) Draw line segments between points A and B, B and C, C and D, and D and A. Give the most precise name of the figure formed.

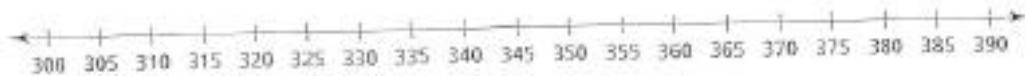
C) Find the area, in square units, of the figure you drew on the graph paper.
- A) On graph paper, graph and label $\triangle ABC$ with vertices A(2, 1), B(9, 1) and C(5,5).

B) On the same graph, Graph and label DE with vertices D (2,-1) and E (9,-1).

C) Using DE, draw a second triangle that is congruent to $\triangle ABC$ and is the reflection of $\triangle ABC$ in the x-axis.
- Heather is making a pair of bookends from a clay cone. She will cut the cone in half, as shown below. Draw the polygon that shows the shape of the cut face of each half and give the name for the figure you drew.



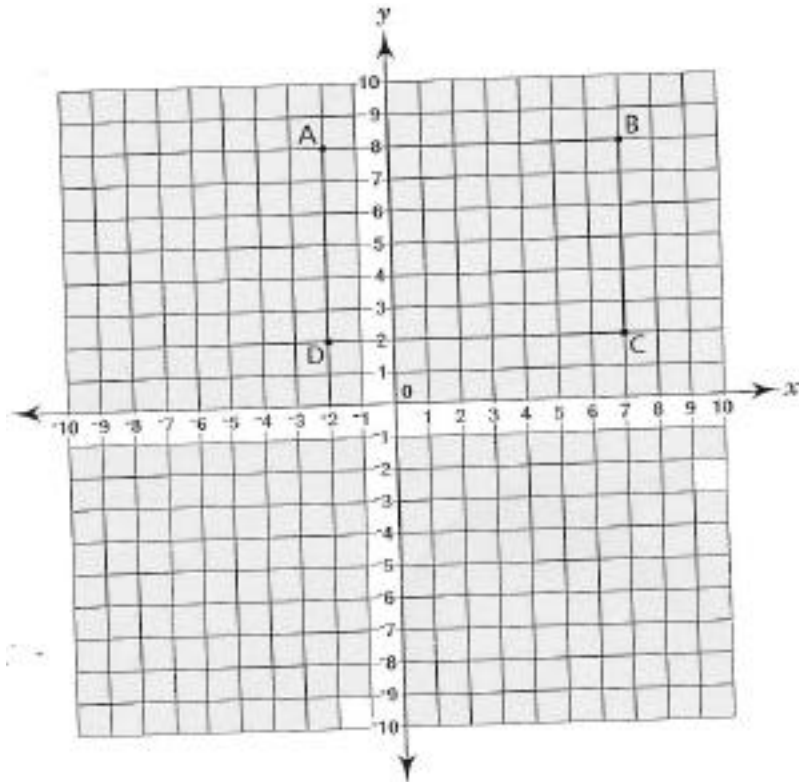
- An advertisement for a new car states that the car travels 17 to 20 miles for every gallon of gasoline. The fuel tank on the car holds 18 gallons of gasoline. On the number line below, represent the possible range for the distance this new car could travel on one full tank of gasoline.



5) Roberto is mapping an area in the desert and discovers an old abandoned mine. It is located within rectangle ABCD shown on the map grid below. Roberto determined the coordinates of 3 different landmarks to help find the entrance to the mine. The landmarks are at points E (6,4), F (2,3), and G (2,7).

A) On the grid, plot and label points E, F, and G.

B) The mine entrance is located at the intersection of AE, BF, and CG. Draw the line segments on the grid to determine the location. What are the coordinates of the entrance to the mine?



6) Miguel wants to make a kite with a shape similar to the one shown below. The length of the sides of the new kite will be 15 times larger. (Use the centimeter side of your ruler to help you solve this problem.)

A) What would be the perimeter in centimeters of the new kite?

B) Would the angles of the new kite be larger, smaller, or the same as the angles of the kite pictured below? Explain your answer completely.



Measurement

1. Ahmir is doing a project on snowfall. The table below shows snowfall data for six New York cities in November 1995.

CITY	TOTAL SNOWFALL (in inches)	DEPARTURE FROM NORMAL (in inches)
Albany	5.8	+0.8
Binghamton	34.1	+26.6
Buffalo	15.4	+5.5
New York City	2.9	- 0.4
Rochester	23.4	+17.0
Syracuse	34.2	+25.1

- A) What was the mean snowfall for all of these cities in November 1995?
B) Using the information in the table above, calculate the normal November snowfall, in inches, for Syracuse.

2. Marina studied the number of people using the escalator and the elevator in the mall for one hour per day for 6 days. Her figures are listed in the table below.

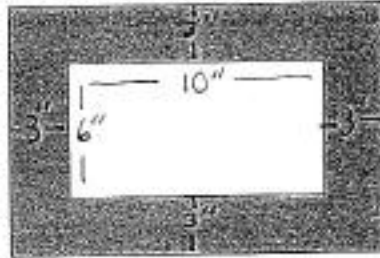
DAY	NUMBER OF PEOPLE USING ESCALATOR	NUMBER OF PEOPLE USING ELEVATOR
Monday	30	12
Tuesday	23	15
Wednesday	19	17
Thursday	17	11
Friday	33	23
Saturday	40	24

- A) Using the information from the table, create a bar graph showing the number of people using the escalator and the elevator for one hour per day over the 6-day period. Be sure to:

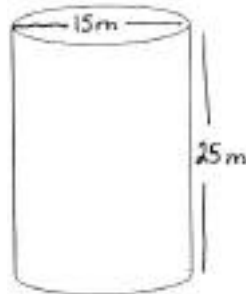
- Title your graph
- Label the axes
- Use appropriate and consistent scales
- Accurately graph the data
- Use an appropriate key

- B) Make one conclusion based on the information from Marina's study and write it in a complete sentence.

3. A baker is decorating the top of a cake with a 3-inch border of chocolate frosting surrounding a 6-by-10 inch rectangle of vanilla frosting. A diagram of the top of the cake is shown.



- A) What is the total area, in square inches, of the chocolate frosting border?
- B) On a second cake of the same size and shape, the baker makes a 5-inch by 9-inch rectangle of vanilla frosting, and then covers the rest of the top of the cake with chocolate frosting. How many more square inches of chocolate frosting are on top of the second cake than are on top of the first cake?
4. A large water tank in the shape of a right circular cylinder stands 25 meters high and measures 15 meters in diameter.



- A) What is the volume of the water tank in cubic meters? (Use 3.14 for π)
- B) For every cubic meter, the tank can hold 1,000 liters of water. What is the maximum number of liters of water the tank can hold?

Uncertainty

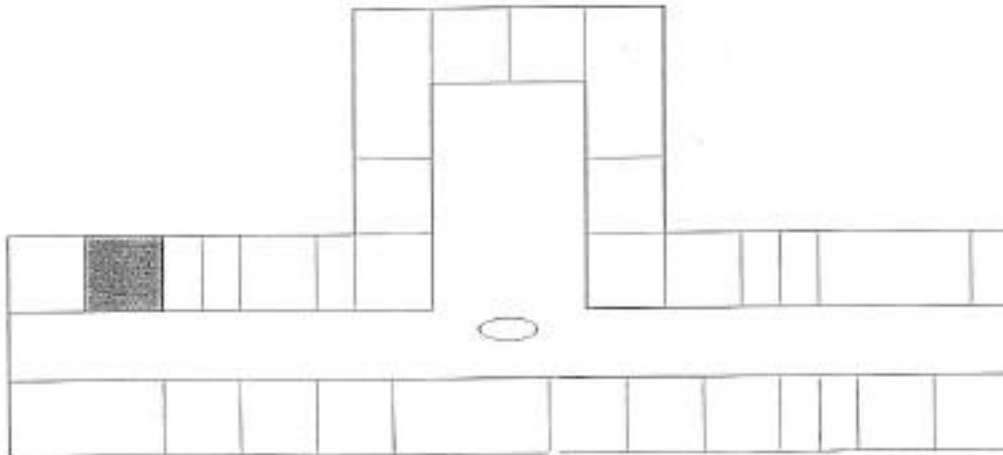
1. A coin collector has a box that contains United States quarters. The quarters have dates from 1954 to 1959, as shown in the table below.

Date	Number of Quarters
1954	19
1955	7
1956	6
1957	8
1958	5
1959	11

If the coin collector chooses one quarter from the box without looking, what is the probability that the quarter selected is dated 1955, 1956, or 1957?

2. Shown below is a floor plan of the west wing of the shopping mall. The store that is shaded represents approximately 1,275 square feet. ESTIMATE the total square footage of all the stores in this wing of the shopping mall. Then explain the process you used to determine your estimate.

WEST WING OF THE SHOPPING MALL



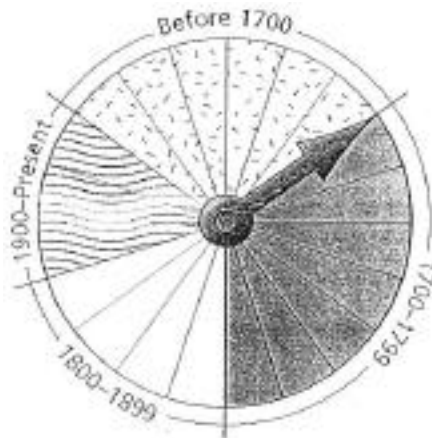
3. Joshua made \$650.00 while working last year doing yard work for various people. He decided to make a graph to record how he spent his money. The graph is shown below. Estimate the total amount of money Joshua spent on CD's last year. Explain in words how you determined your estimate.

HOW I SPENT MY MONEY



4. The “Wheel of History” game uses a spinner divided into 20 equal sections, each labeled with a time period in American history. A team member spins the spinner and is asked a question about the time period the spinner lands on.

- A) What is the probability that the spinner will land on the time period 1800-1899?
B) The history department is planning to write 60 questions for the game. Based on the probability you gave in Part A, how many questions should be written for the period 1800-1899?



Patterns/Functions

1. The house wares store at the shopping mall is promoting a new floor scrubber. The chart below shows how much money you save depending on how many floor scrubbers you buy. The amount of money you save increases with the number of floor scrubbers you buy.

Number Bought	Amount of Savings
3	\$5.00
6	\$10.00
9	\$15.00
12	\$20.00

You purchase 21 floor scrubbers, spending a total of \$299.95 after the discount. What was the price, p , of each floor scrubber before the discount?

2. Maya's class is selling ice cream cones for \$0.75 each at the spring fair. The class spent \$60.00 for the ice cream and cones. The equation below can be used to find the amount of profit the class will make.

$$\text{Profit} = \$0.75 \times \text{number of ice cream cones sold} - \$60.00$$

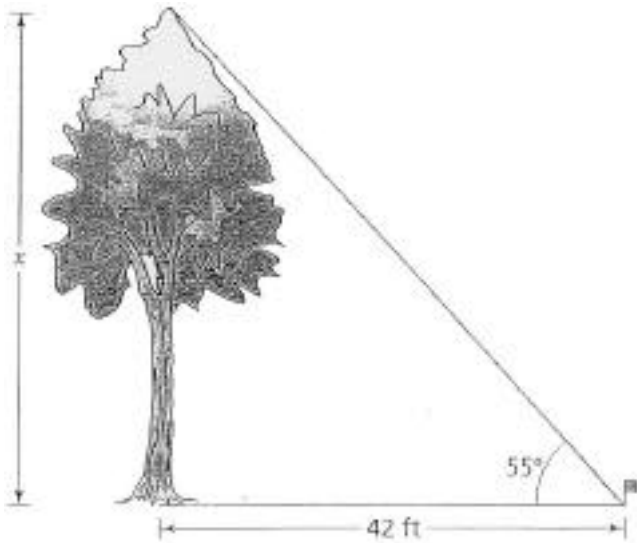
Use the equation above to find how many ice cream cones Maya's class must sell to make a profit of \$30.00.

3. The school planning committee wants to rent as many tents as they can for the spring fair. The committee has received from three rental companies the following information.
 - Rent-a-Roof charges \$325 for the first 3 tents, and \$125 for each additional tent. There is no charge for delivery and setup.
 - Parties-R-Us has a one-time charge of \$100 for delivery and setup, plus \$100 for each tent.
 - Tents-'n-More charges \$125 for each tent. There is no additional charge for delivery and setup.

The committee has \$600 to spend on tents.

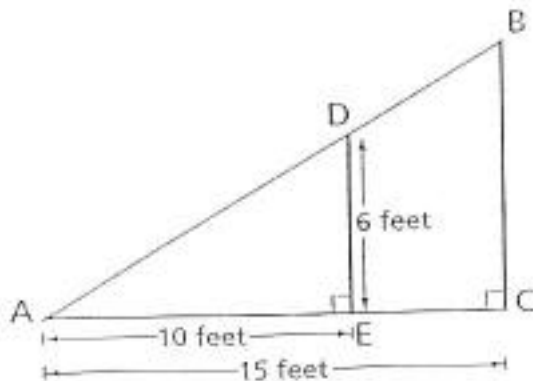
- A) What is the maximum number of tents it can rent?
- B) What company will give the best price for the maximum number of tents? Explain your answer.

4. Tyrone cut a quadrilateral from a piece of cardboard. One pair of opposite sides is parallel but not congruent, and the other pair of opposite sides is congruent but not parallel. Draw the shape of the quadrilateral he cut.
- What is the name of the quadrilateral that you drew above?
 - Draw a quadrilateral in which both pairs of opposite sides are parallel and congruent.
 - What is the name of this new quadrilateral?
5. An engineer for an electric company is calculating the height of a tree. She placed a flag on the ground 42 feet from the base of the tree and used her surveying equipment to find that the angle of elevation from the flag to the top of the tree is 55° . Find the height, x , of the tree to the nearest foot.



6. In the figure below $\triangle ADE$ is similar to $\triangle ABC$.

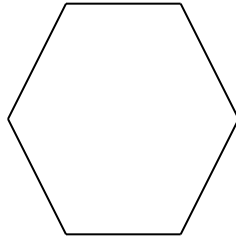
- What is the length in feet of BC ?
- What is the length in feet of AB ?



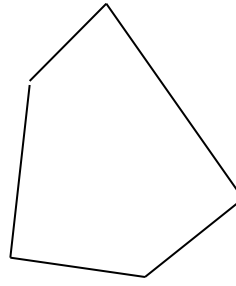
7. Use the centimeter side of your ruler and your protractor to help you solve this problem. Look at the polygons below.



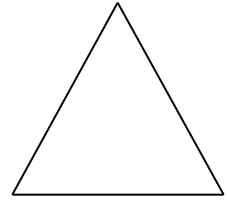
A



B

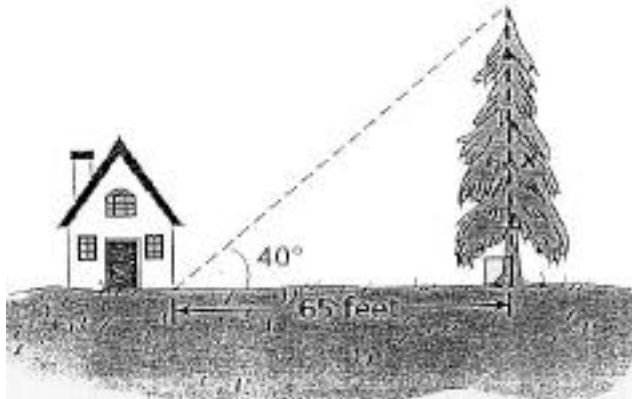


C



D

- A) Which of the polygons above are regular polygons?
B) Explain in words how you know a polygon is regular.
8. A tree, which is in danger of falling, stands 65 feet from a house. The angle of elevation from the bottom of the house to the top of the tree is 40° . If the tree should fall directly toward the house, will it hit the house? Show all mathematical work necessary to make your conclusion.



Thematic Questions

After Christmas

After Christmas students were asked to hand in a list of five ways they had used math during their Christmas holiday. I then gave them a list of five ways that I had used math and a problem that applied to each of those.

1. When wrapping shirt boxes I found that a strip of paper 22 inches wide would wrap one box. Each roll of wrapping paper was 6 feet long. If I had 17 shirt boxes to wrap, how many rolls of paper did I need to buy?
2. My recipe for sweet potatoes and apples calls for the following ingredients:
 - 2 sweet potatoes
 - 4 apples
 - 4 Tablespoons brown sugar
 - 6 Tablespoons butter
 - 1/2 stick butter

This recipe serves 4 people. I had 14 people for Christmas dinner. Change the recipe to reflect the number of people. Explain in words how you did this.

3. My turkey for Christmas dinner needed to cook for 3 hours and 45 minutes. Then I had to take it out of the oven and cook my other items 30 minutes, after first raising the temperature of the oven, which took 10 minutes. If I served dinner at 4:00 p.m., what time did I put the turkey into the oven?
4. When I shopped at Price Chopper for Christmas dinner the bill came to \$110.17. I then turned in \$17.35 in coupons, which were doubled. Finally, my Advantage Card gave me an additional savings of \$9.48. What was my final cost?
5. I received a \$25 gift certificate to JC Penney's as a Christmas gift. When I went shopping, all the things I wanted were on sale at 40% off. I picked out \$38.00 worth of items.
 - A) What was the discount cost of these items?
 - B) How much did I get back or have to pay if the tax rate was 5%?

Dr. Martin Luther King's Birthday



1. Martin Luther King's birthday is January 15. He would have been 73 years old this year. He became a minister at the age of 17 and spent much of his life preaching non-violence. During the fifties and early sixties he became very well known as a leader of the civil rights movement. He was assassinated on April 4, during my freshman year of college.

 - A) In what year was Martin Luther King born?
 - B) I graduated from high school in June, 1967. How old was Martin Luther King when he died?
 - C) For approximately how many years was he a minister?
2. The temperature this morning was -11 degrees in Grand Forks, North Dakota, where my son lives. The temperature here was the additive inverse of this?

 - A) What was the temperature here?
 - B) Define what we mean by additive inverse.
 - C) What do we call the multiplicative inverse of a number?
3. Shellie wants to make a bar graph showing the number of days it rained and the number of days it did not rain in the last 300 days. She finds that it rained 20% of the time. She decides to label the y-axis using 20-day intervals.

 - A) How many days did it not rain?
 - B) What is the height of the bar she must draw to represent "days with no rain"?
4. In the following series of equations a, b, c, and d represent different integers. If $d = 5$, find the values of a, b, and c. Show all the work you do, or name the properties you use.

$$a + b = b$$

$$b + c = 0$$

$$b \cdot d = d$$

5. Arrange the following numbers in order from least to greatest. Explain in words how you did this.

$$25\%, 0.22, 2.5 \times 10^{-1}, 28 \times 10^{-2}$$

Jewelry Heist

The following are stories about Abdul's Jewelry Store. These were written many years ago. Each can be solved in many ways. Don't let the language or the stories confuse you. They are not as hard as they may seem. Solve these, showing all of your work for each, and do each one on a separate sheet of paper. Please do not write on the backs of the papers.

1. Two thieves stole into Abdul's jewelry shop. Some of them were armed and some were unarmed. The armed ones were those of senior rank. They stole a bag of fifty-six pearls. When it came time to divide them up, each senior robber took 6 pearls, each junior robber got 5. How many of the robbers were senior?
2. One day a man bought 59 jewels to Abdul. Some were emeralds and some were rubies. The emeralds were carried in bags, nine to a bag, whereas the rubies were carried four to a bag. How many of the jewels were rubies?
3. Another day a customer brought into the shop of Abdul the Jeweler six chains, each of which had 5 links. He wanted the 6 chains to be joined into one large circular chain and inquired as to the cost. Abdul answered, "Every link I cut open and close costs one piece of silver." How many pieces of silver are required for the job?
4. One night a thief stole into Abdul's shop. He came upon a pile of diamonds. His first thought was to take them all, but then his conscience bothered him, and he decided to content himself with only half. He took half of the diamonds and started to leave and then thought, "I'll take one more. And so he left the shop having taken half the diamonds plus one. Strangely enough, a few minutes later, a second thief entered the shop and took half the remaining diamonds plus one. Then a third thief entered the shop and took one half the remaining diamonds plus one. A fourth thief entered the shop later that night and took one half the remaining diamonds plus one. Finally a fifth thief entered, but took no diamonds since they were all gone. How many diamonds were in the pile to start with?

Florida

1. My mother and sister spent last week in Florida. They went to Sea World one day. The admission price was \$59.00 each, but my mother got a 10% discount on her ticket. Once inside they rented a wheel chair for my mom, which cost \$7.00. Their lunches cost \$12.50 each, but again my mother got the 10% discount. They had decided to split all costs equally. How much did they each have to pay?
2. While in Florida they visited my brother. I knew they were going to be there and tried to call, but my brother's area code had changed and I didn't know what it was. Area codes are 3-digit numbers. The first number is never a 0 or 1. The second and third numbers can be any digit. How many different area codes could there be?
3. My brother is the director of an art museum in Lakeland, Florida. They sell prints of famous pictures. The price of the prints varies according to how many you buy. The table below shows how the price list begins. Using this table, determine how much it would cost to buy 18 prints. Show your work or explain how you got your answer.

Number of Prints	2	4	6	8	10
Cost per Print	\$136	\$134	\$131	\$127	\$122

4. My niece, Natalie, goes to a school of performing arts in Lakeland. This school teaches not only the core courses but also specialized music and art courses. She must take 22 credits in core courses and 9 credits in music to graduate. In order to do this the students must attend summer school. During the regular year the students can take 6 credits and during summer school they can take 3. Determine how many years she must attend summer school. Show your work or explain how you got your answer.

Cross Curricular Questions

Science

1. Bacteria are single celled organisms. They reproduce by dividing in two. If you start with one bacteria that reproduces every 20 minutes, how many will there be after 2 hours? (Hint: Make a table)
2. A mole of atoms is defined as the number of atoms in one-gram atomic mass of an element. This number is defined as being 6.02×10^{23}
 - a) If this number is written in standard notation, how many zeros will there be between the decimal point and the six?
 - b) If the atomic mass of oxygen is 16, how many atoms of oxygen will be in 4 grams of oxygen?
3. To launch a new satellite, engineers need to design a fuel tank cylinder that will hold at least 380,000 cubic feet of fuel. The fuel cylinder must have a diameter of 32 feet. What is the minimum height the fuel tank must be in order to hold all the fuel?
4. During the 180 days of school in one New York school students kept track of the weather every day. They found that it rained 30% of those days and snowed another 25% of the time. How many days during that time period did the students see no precipitation?
5. A person's normal heart rate is found to be 72 beats per minute. How many times will their heart beat in during the year 2001?

Social Studies

1. The number of immigrants to the United States in 1900 was 448,572. This number increased to 1,041,570 in 1910. What was the approximate percentage of increase during this time period?
2. While reconstructing the travels of Lewis and Clark students used the map below.



Using the given scale (1/4 inch = 100 miles) and the route traced onto the map, approximately how many miles was their journey?

3. There are currently 435 members of the House of Representatives. A two-thirds margin of the members present is needed for a new bill to be passed.
 - a) Write two-thirds as a percent.
 - b) How many members of the House of Representatives must approve a bill if 15 members are absent?
4. The bombing of Pearl Harbor on December 7, 1941 caused the United States to enter World War II. The Japanese surrendered on August 14, 1945. During this time period the United States fought the war on two fronts: in Europe (including parts of northern Africa) and in the Pacific. It is estimated that the economic cost of the war to the United States was \$341 billion dollars. What was the approximate cost of the war per month to the United States?
5. The social studies class wants to earn money to go to the museum. The entry cost to the museum is \$6.50 per person. The cost of renting buses is \$12.00 per bus per hour and \$1.50 per mile. Each bus can hold 45 people and there are 120 people going on the trip. The distance to the museum is 35 miles each way and the trip will take a total of 6 hours. What is the total cost for the class to attend the museum?