An application of eigenvalue methods to structural domain analysis

FREDERICK W. VOLLMER *Department of Geological Sciences, State University of New York, New Paltz, New York 12561*

ABSTRACT

The subdivision of a geologic map into structural domains involves the location of regions in which the rock fabric has certain geometrical characteristics; typically, foliation data should share a common axis. The location of such domains involves working interactively with map and equal-area projections of the data set, a tedious and often subjective process. Eigenvector methods can quantify this type of analysis. A set of four eigenvalue-based indexes assists in discriminating among fabric distributions, particularly between strong and weak cylindrical distributions. These indexes form the basis of a triangular diagram for distinguishing among point, girdle, random, and cylindrical fabrics. A domain search proceeds by subdividing the data set and attempting to maximize the total cylindricity, or other characteristic. The method has been applied to a set of foliation data from a fold nappe in the Western Gneiss Region of the Norwegian Caledonides. The resulting domains define a late fold that refolds the recumbent nappe. Lineation and fold-axis data from the area support the results of the domain search. This method may be valuable in other areas of complex geometry.

INTRODUCTION

In areas of complex geologic structure, for example where superposed folds are present, the location of cylindrical fold domains is in many cases required for structural analysis (Turner and Weiss, 1963, p. 169–185; Ramsay, 1967, p. 518–555). These domains have the characteristic that poles to bedding, or other foliation planes, lie perpendicular to a single axis. If such domains can be located, structures within them can be projected downplunge to provide a three-dimensional interpretation of the regional structure.

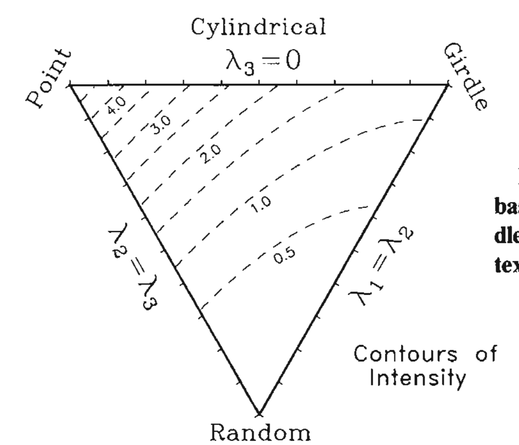

Figure 1. Triangular fabric diagram based on the indexes *P* (point), *G* (girdle), and *R* (random), defined in the text. Contours are of equal intensity, *I*.

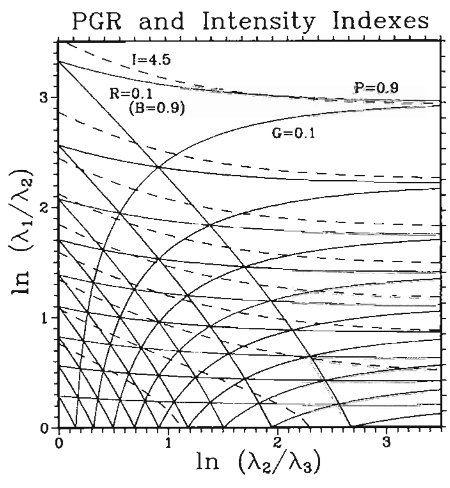

Figure 2. Fabric plot of Woodcock (1977), showing contours of the *P*, *G*, *R*, and *B* (1 − *R*) indexes (solid) and intensity, *I* (dashed). The four indexes range from 0 to 1; intensity ranges from 0 to 5.

Such an analysis may fail if the folds are inherently noncylindrical, although one can attempt to locate domains that have other, noncylindrical, characteristics. A second limitation is that the three-dimensional geometry of the domain boundaries is difficult to determine because they can in most cases be mapped only at the Earth's surface. As in other geometrical methods, multiple deformation and transposition may limit the amount of interpretable information within the rock body (Hobbs and others, 1976, p. 371–375; Williams, 1985).

Despite these restrictions, domain analysis remains a useful method for the analysis of complex structures. Whitten (1966, p. 358–476) and Ramsay and Huber (1987, p. 475–504) have given examples of this type of analysis, including classic studies by Weiss and McIntyre (1957) and Ramsay (1958). Recently, Langenberg and others (1987) used cylindrical fold domains to construct computer-generated cross sections through the Morcles Nappe.

This paper describes an eigenvalue method for the location and analysis of such structural domains. A sample application is given from a multiply folded nappe in the Western Gneiss

Geological Society of America Bulletin, v. 102, p. 786–791, 8 figs., June 1990.

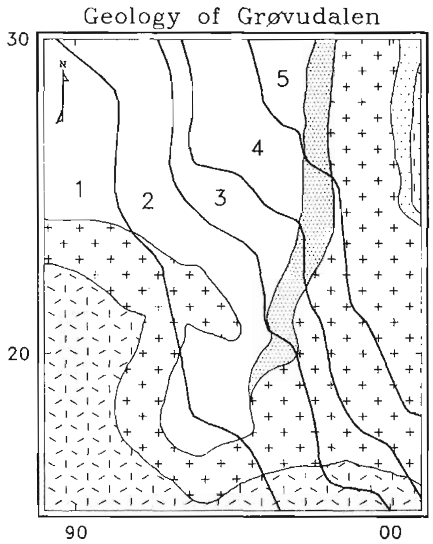

Figure 3. Geology of the Grøvudal area, showing the principal lithologic contacts and the five domain boundaries (heavy solid lines) defined using the method described in the text. The coordinates are 10-km UTM coordinates in grid zone 32V-MQ. The rock units are high-grade metasediments and orthogneisses (Krill, 1980; Vollmer, 1985, 1988).

Region of the central Norwegian Caledonides. The use of this methodology is suggested for other areas of structural complexity, where the simple examination of map patterns does not reveal obvious structural domains, or where an unbiased search is desired. The method is particularly applicable to the computer analysis of fabric data. A triangular plot is defined for the graphical treatment of fabric data.

METHODS OF DOMAIN ANALYSIS

Turner and Weiss (1963, p. 175–185), Ramsay (1967, p. 518–555), Hobbs and others (1976, p. 372–373), and Ramsay and Huber (1987, p. 475–504) discussed structural domain analysis and described a number of methods for locating regions of cylindrical folds. These include the examination of fold axis, lineation, and bedding orientations; cleavage and secondary foliation relationships; and the symmetries of fold interference patterns. Hobbs and others (1976, p. 373) suggested that the "experienced geologist can generally recognize subareas during the process of mapping." This, however, may be difficult where domain boundaries are gradational or where minor folds are rare. In those cases, the analysis normally proceeds by "trial and error" (Turner and Weiss, 1963, p. 175). This involves the initial drawing of likely domain boundaries, plotting of the domain data on equal-area projections to check for cylindricity, and repeated redefining of the boundaries to improve the fit.

A number of problems exist with a trial-and-error approach to domain analysis. First, it is tedious, involving the iterative plotting of large amounts of data. This tends to reduce the number of solutions explored and decreases the likelihood of finding the "best" solution. Second, where gradational changes or local variations (noise) make the boundaries indistinct, it may be necessary to define them statistically. Finally, the definition of a "best" solution is not clear. Although different types of information must be used when subdividing a map into domains, it is desirable to have a measure of goodness of fit.

Hence, there is a need for a method to be used where simple examination of the map data is not sufficient, as in the study of the geometric form of a refolded nappe in central Norway by Vollmer (1985). In that study, a computer-aided method was used to locate cylindrical fold domains by minimizing the angular deviations of foliation poles from domain eigenvectors (Vollmer, 1985, 1988). A more general method is presented herein, in which domains are located by maximizing an index based on the eigenvalues of orientation data.

EIGENVECTORS OF ORIENTATION DATA

The orientation of a line, or the pole to a plane, can be defined by a column vector of direction cosines, $\mathbf{L} = (l, m, n)'$, where l, m, and n are the direction cosines with respect to the axes north, east, and down, respectively. The eigenvectors of a set of N orientation data are then calculated from the matrix

$$\mathbf{M} = \Sigma\, \mathbf{L}_i\, \mathbf{L}_i'$$

where $\mathbf{L}'$ is the transpose of $\mathbf{L}$, and summation is from $i = 1$ to N (Scheidegger, 1965; Watson,

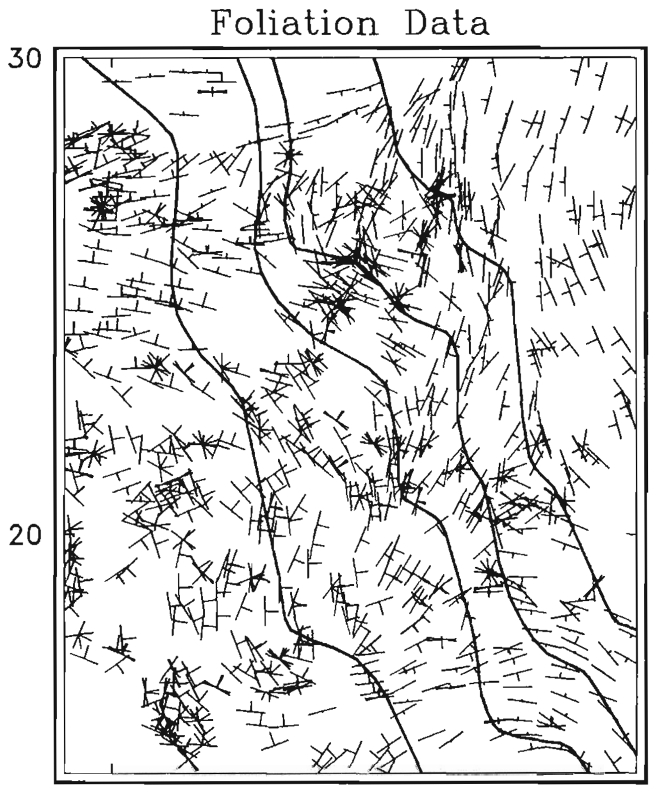

Figure 4. Data set of 957 foliation measurements from the Grøvudal area, used in the example domain analysis. The length of each dip tick is proportional to the cosine of the dip, giving its horizontal projection. Domain boundaries and coordinates are the same as in Figure 3.

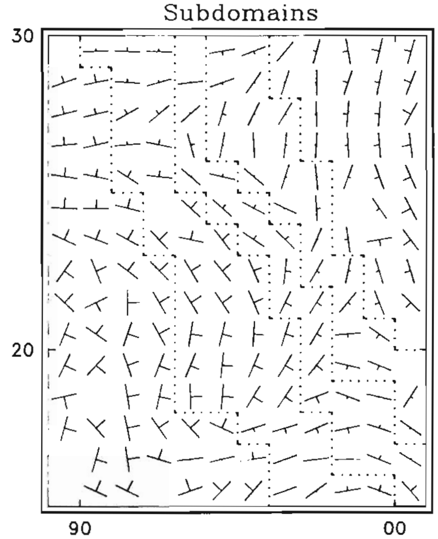

Figure 5. Eigen-foliations for the Grøvudal subdomains. Each foliation is perpendicular to the maximum eigenvector of the data within a 1-km-square subdomain. The dotted lines are the domain boundaries before final refinement. The dip ticks are projected to horizontal.

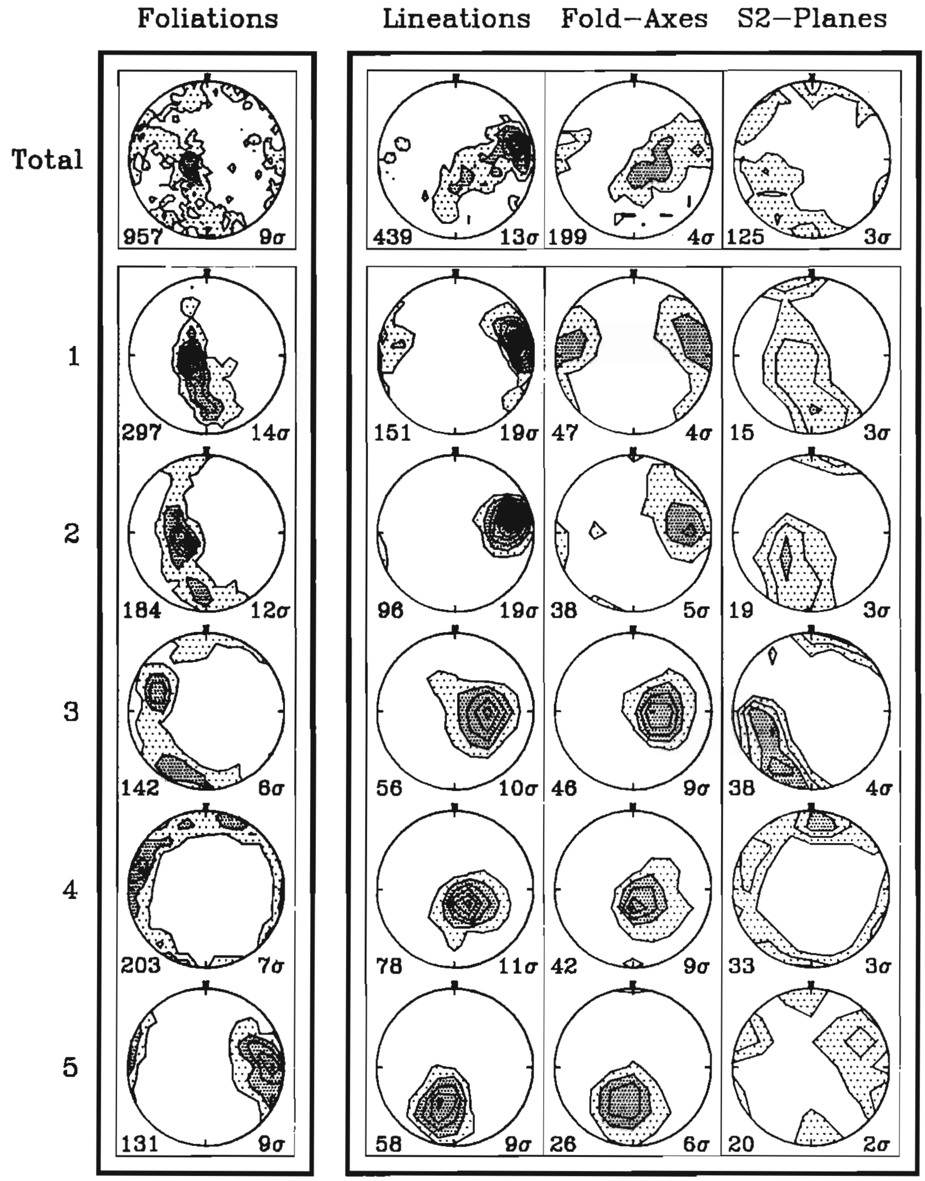

Figure 6. Lower-hemisphere equal-area projections of data from the Grøvudal area, contoured using Kamb's (1959) method. Nets 1–5 correspond to the domains shown in Figure 3. "Foliations" are the dominant, nonprimary, layering. "Lineations" include stretching and mineral lineations. "Fold-Axes" are from folds in the dominant foliation. "S2-Planes" include later foliations and fold axial planes. The contour interval is 2σ, except for the S_2 planes, which are contoured at 1σ. Light stipple begins at 1σ, and heavy stipple begins at 3σ. The number of data points is indicated at the lower left of each net, and the maximum concentration is indicated at the lower right.

1965). The three eigenvalues, λ_1, λ_2, and λ_3, are calculated for this matrix such that

$$\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3$$
$$\lambda_1 + \lambda_2 + \lambda_3 = N$$

The maximum eigenvector corresponds to the "mean" or best-fit axis of the data (Scheidegger, 1965), and the relative magnitudes of the eigenvalues give a measure of the strength and symmetry of the fabric (Watson, 1966):

point	$\lambda_1 > \lambda_2 \approx \lambda_3$
girdle	$\lambda_1 \approx \lambda_2 > \lambda_3$
random	$\lambda_1 \approx \lambda_2 \approx \lambda_3$

An eigenvalue analysis is most useful for fabrics that can be described with respect to the axes of an ellipsoid, with orthorhombic or unimodal axial symmetries. Confidence regions about each eigenvector can be calculated (Bingham, 1974; Mardia and Zemroch, 1977).

CYLINDRICITY

A measure of the cylindricity of a set of fabric data is desirable for domain analysis. Langenberg and others (1987) used an F-test to determine whether statistical differences existed between adjacent cylindrical domains. Charlesworth and others (1976) and Kelker and Langenberg (1988) discussed this test and other related statistical tests. Several indexes have been proposed that relate to cylindricity. Woodcock (1977) suggested a parameter, C, which ranges from 0 to positive infinity,

$$C = \ln(\lambda_1/\lambda_3)$$

as a measure of fabric strength. Large values of C indicate point, girdle, or intermediate distributions. Lisle (1985) preferred the intensity, I, based on the uniformity statistic, S_u, of Mardia (1972). I ranges from 0 to 5 and is defined as

$$I = 7.5\,[(\lambda_1/N - 1/3)^2 + (\lambda_2/N - 1/3)^2 + (\lambda_3/N - 1/3)^2] = S_u/N$$

For the present purposes, a set of parameters was desired that distinguishes among point, girdle, and random distributions, and the following

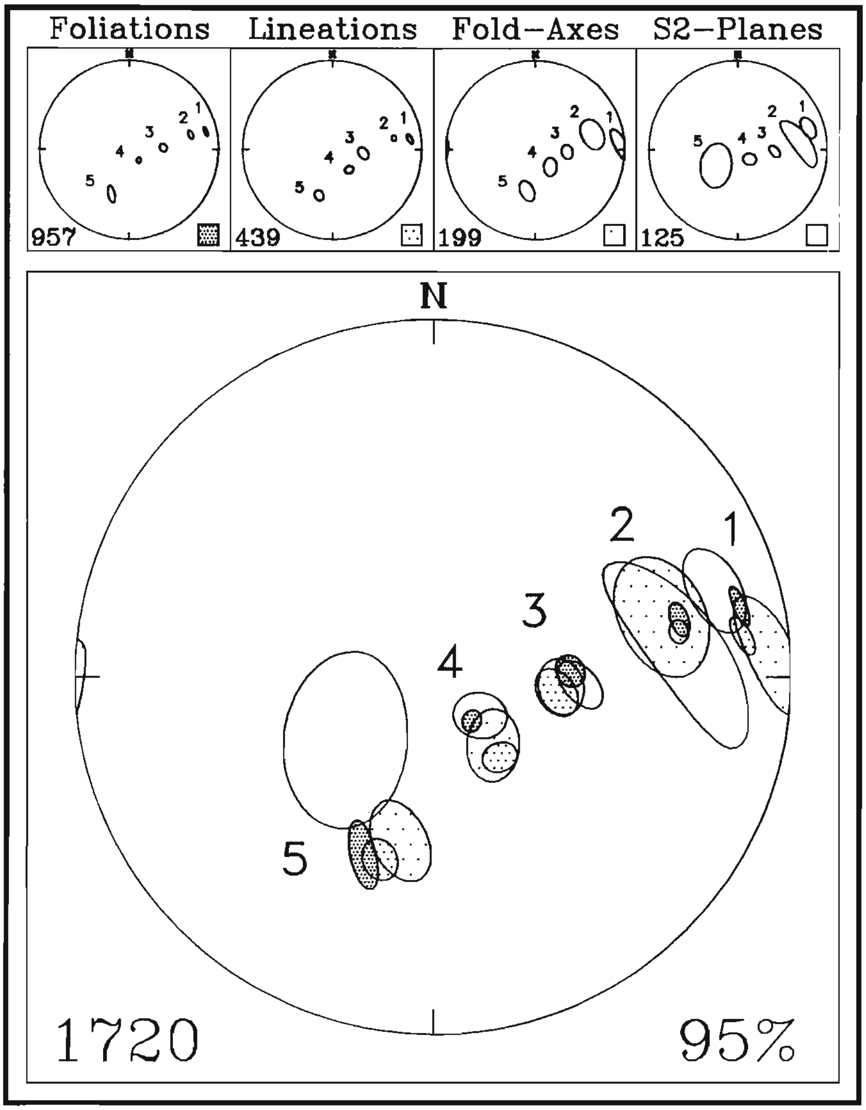

Figure 7. Synoptic plots of the Grøvudal data, showing the 95%-confidence regions of foliation minima and lineation maxima from each of the five domains. The maxima and minima from each domain define approximately equally spaced points along a small circle, suggesting refolding of all fabric elements about a northwest-trending axis.

graphical-based approach was developed. The three end-member fabric types have eigenvalues $[\lambda_1\ \lambda_2\ \lambda_3]$ as follows.

$$\begin{aligned} \text{point} &= [N \quad 0 \quad 0\ ] \\ \text{girdle} &= [N/2 \quad N/2 \quad 0\ ] \\ \text{random} &= [N/3 \quad N/3 \quad N/3] \end{aligned}$$

If we give these three end-member distributions equal weight, then three indexes, P, G, and R, can be defined as

$$\begin{aligned} P &= (\lambda_1 - \lambda_2)/N \\ G &= 2(\lambda_2 - \lambda_3)/N \\ R &= 3(\lambda_3)/N \end{aligned}$$

to measure the relative contributions of each end member. These indexes range from 0 to 1 and have the property that

$$P + G + R = 1$$

A fourth index, of cylindricity, B, is defined as

$$B = P + G$$

which equals 0 for a random or uniform distribution, and 1 for either a perfect girdle or a perfect point distribution. The index B thus measures how nearly perpendicular a set of data is to an axis without distinguishing between point and girdle distributions. This makes a set of data taken from the limb of a fold equivalent to one from the hinge.

TRIANGULAR FABRIC PLOT

The three indexes P, G, and R may be taken to define a triangular plot (Vollmer, 1989) analogous to that proposed by Woodcock (1977) in which $\ln(\lambda_1/\lambda_2)$ is plotted versus $\ln(\lambda_2/\lambda_3)$. Woodcock's fabric plot has two axes corresponding to cluster and girdle distributions, with values ranging from 0 to infinity. The triangular *PGR* plot differs in that it is closed and has three end members. It is based on the differences between the eigenvalues, rather than their ratios.

Contours of intensity, I, on the *PGR* plot (Fig. 1) can be compared with contours of P, G, R, B, and I on the plot of Woodcock (1977), shown in Figure 2. On the *PGR* plot, contours of P, G, R, and B are equally spaced straight lines parallel to the three axes. The cylindricity index, B, increases linearly from 0 at $R = 1$, to 1 at the P–G join. The spacing of the intensity contours on the *PGR* plot makes it most useful for fabrics of low intensity, typical of geologic fabrics. An expanded partial plot could be used for strong fabrics.

DOMAIN SEARCH

A search procedure locates domain boundaries by maximizing a suitable fabric index. The method is illustrated in an example from the Grøvudal area (Fig. 3) within the Western Gneiss Region, central Norway, where data from a refolded nappe were collected (Vollmer, 1985). Whereas the example maximizes the cylindricity of planar data, analogous methods can maximize other statistical parameters.

First, the area was subdivided into *subdomains*, using a grid of boundaries. A total of 957 foliation measurements were available throughout the Grøvudal area (Fig. 4), and a grid spacing of 1 km was used, yielding 12 × 15 subdomains (Fig. 5). Outcrop coverage varied, and ten of the subdomains remained empty. The size of the subdomains was chosen to be as small as possible without creating an undue number that were empty.

For each subdomain, the matrix $\mathbf{M}_{ij}$ was calculated, where i and j refer to the subdomain row and column. Figure 5 shows the foliation data projected as a tensor field, similar to Scheidegger's (1965) method for displaying orientations. An *eigen-foliation* for each subdomain designates the plane perpendicular to the maximum eigenvector derived from the set of foliation poles. A grid of orientation data was thus obtained in the form of the subdomain matrixes $\mathbf{M}_{ij}$, which retain information from all the data

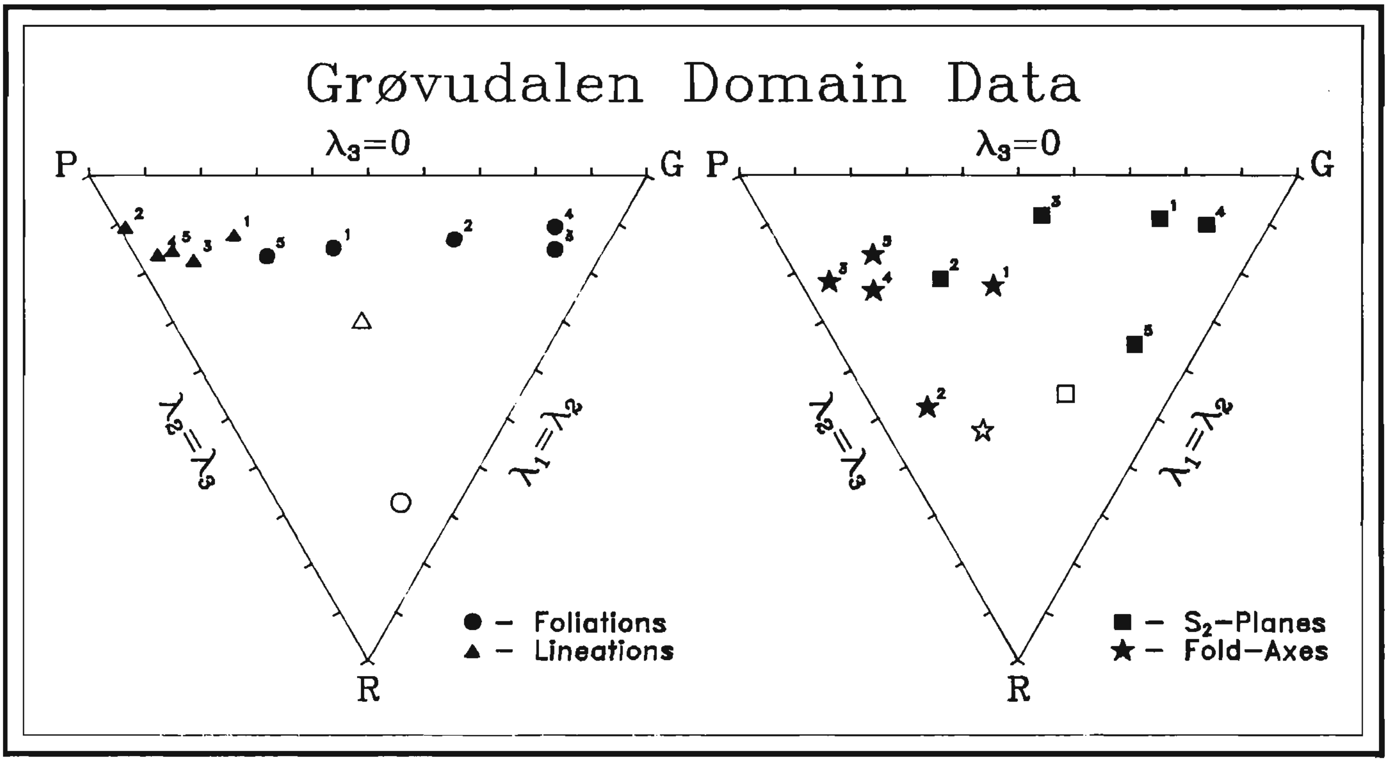

Figure 8. ***PGR*** **fabric plots of the combined Grøvudal fabric data (open symbols) and the domainally separated data (solid symbols, 1–5). See Figure 6 for the corresponding equal-area nets.**

points rather than combining them into a single average value. The set of all subdomains was then divided, by inspection, into n initial domains. Five initially rectangular domains were chosen, but in general, n, the number of domains, depends on both the number of subdomains and the complexity of the area.

The search proceeded by calculating the domain matrix $\mathbf{M}_k$ for each initial domain, k, as the sum of its constituent subdomain matrixes. The eigenvectors of each domain matrix $\mathbf{M}_k$ were then used to determine a fabric index. In this case, the quantity maximized was the sum of the number of data points within a domain, N_k, times the domain cylindricity index, B_k,

$$Z = \Sigma N_k B_k$$

summed over $k = 1$ to n.

The search continued by randomly selecting individual subdomains at domain boundaries for potential exchange with the neighbor domains. First, however, it was assured that exchange would leave both the host and the neighbor domain continuous. The geometry was also checked to avoid deep embayments into domains. When a subdomain was found to be exchangeable it was determined whether the exchange would increase Z. The matrixes $\mathbf{M}_k$ of the host and neighbor domains were recalculated, Z was redetermined, and the exchange was made only if Z would increase.

This was repeated until the domain boundaries remained static and the resulting domain geometry then represented one of potentially several possible solutions. The search was repeated with different initial domains until an optimal solution emerged. In the example, it was found that searches with different initial configurations tended to converge toward the same general pattern of elongate, north-northwest–trending domains. The best solution had Z = 814, and an average of B = 0.85 (Fig. 5).

The data set was then edited by finding for each datum the domain number to whose axis it is most nearly perpendicular, and plotting that number on the map. Domain boundaries were then redefined to enclose as many "stray" points as possible. This smoothed the domain boundaries and increased the overall Z. Boundary adjustments may be repeated iteratively. This editing resembles the procedure used by Vollmer (1985). The domain boundaries shown in Figures 3 and 4 were obtained in this way. Editing increased the total cylindricity Z to 824, and average B to 0.86.

DISCUSSION

The structural domain boundaries located in the sample area are shown in Figures 3 and 4. Note that these boundaries are in no obvious way correlated with the geologic contacts and that the initial foliation measurements do not in any simple way suggest structural domains.

To illustrate the results, domainally separated data from the Grøvudal area are shown on contoured equal-area projections (Fig. 6), synoptic equal-area projections (Fig. 7), and *PGR* fabric plots (Fig. 8). The foliations show well-defined girdle and cluster patterns, in marked contrast to the scattered distribution of the combined foliation data (Fig. 6). Similarly, the lineation and fold-axis data show strong clustering within individual domains. As the lineation, fold-axis, and S_2 foliation data sets were not used for the analysis, the strong emerging patterns confirm the geologic significance of the domains.

Note that the foliation minima and lineation maxima from the individual domains are approximately equally spaced along a small circle (Fig. 7). The interpretation is that early east-trending structures in a recumbent nappe were refolded about a northwest-trending axis (Vollmer, 1985, 1988). Thus the structural domains outline a late subhorizontal antiform, slightly overturned to the east (Fig. 3).

The late foliations and fold axial planes (S_2 planes) form the same patterns as do the main foliations (S_1, Figs. 6 and 7) and are thus related to the early recumbent nappe and were also refolded. Although younger, commonly angular, folds and foliations (S_3) exist, they are rare and variable in orientation (Vollmer, 1985).

Figure 8 shows the data on *PGR* diagrams. The combined foliations, which plot at $B = 0.32$, differ strikingly from the domainally separated foliations, which have a range of B from 0.83 to 0.89, spread along the P–G join. The point index for the lineations similarly increases from $P = 0.36$ for the whole area, to a range of P from 0.68 to 0.88 for the domains. The data for fold axes and S_2 planes show similar but less strong increases.

In the Grøvudal area, lineations can be used independently to locate domains because they are statistically parallel to fold axes (Fig. 7). A domain search to maximize the point index, P, for the lineations revealed north-northwest-trending domains that differ little from those defined by the foliations.

The technique presented analyses orientation data more rigorously than do traditional methods; however, a number of subjective choices must still be made during the domain search. These include the subdomain size, the number of domains, and how irregular the boundaries are allowed to be. These choices depend on data density, distribution, and variation. No attempt has been made to take elevation into account.

CONCLUSIONS

The objective subdivision of complex geologic areas into structural domains can be quantified by drawing domain boundaries so as to maximize one of four eigenvalue indexes. The technique joins fabric with map analysis and does not require the use of geologic data other than orientation; it is intended to form only part of a complete structural analysis.

In the Grøvudal area, domain analysis using only foliations revealed a late antiform that refolds an early recumbent nappe. Because several generations of early subparallel folds have been refolded, and because late folds are rare in outcrop, it is difficult to discover this refolding event from fold interference patterns or symmetries. Because abundant lineations are statistically parallel to the early fold axes, however, it was possible to use the lineation data to confirm the existence of the domains established by the foliations.

The *PGR* indexes and the triangular fabric plot allow graphical comparison of fabric shapes and intensities. They are suitable for orthorhombic, LS-type, fabrics. No attempt was made herein to give the indexes statistical significance, but if necessary, tests discussed by Mardia (1972), Charlesworth and others (1976), and Kelker and Langenberg (1988) could be applied.

ACKNOWLEDGMENTS

Comments by Peter Hudleston, Willem Langenberg, Gerhard Oertel, Dan Schultz-Ela, and an anonymous reviewer greatly improved the paper. The procedures outlined herein are implemented in the program Orient, written by the author. The maps and equal-area projections were created using Orient and imported into AutoCAD for final drafting.

REFERENCES CITED

Bingham, C., 1974, An antipodally symmetric distribution on the sphere: Annals of Statistics, v. 2, p. 1201-1225.

Charlesworth, H.A.K., Langenberg, C. W., and Ramsden, J., 1976, Determining axes, axial planes and sections of macroscopic folds using computer-based methods: Canadian Journal of Earth Sciences, v. 13, p. 54–65.

Hobbs, B. E., Means, W. D., and Williams, P. F., 1976, An outline of structural geology: New York, John Wiley & Sons, 571 p.

Kamb, W. B., 1959, Petrofabric observations from Blue Glacier, Washington, in relation to theory and experiment: Journal of Geophysical Research, v. 64, p. 1891–1909.

Kelker, D., and Langenberg, C. W., 1988, The statistical classification of macroscopic folds as cylindrical, circular conical, or elliptical conical: Mathematical Geology, v. 20, p. 717–730.

Krill, A. G., 1980, Tectonics of the Oppdal area, central Norway: Geologiska Föreningens i Stockholm Förhandlingar, v. 102, p. 523–530.

Langenberg, W., Charlesworth, H., and La Riviere, A., 1987, Computer-constructed cross-sections of the Morcles Nappe: Eclogae Geologicae Helvetiae, v. 80, p. 655–667.

Lisle, R. J., 1985, The use of the orientation tensor for the description and statistical testing of fabrics: Journal of Structural Geology, v. 7, p. 115–117.

Mardia, K. V., 1972, Statistics of directional data: New York, Academic Press, 357 p.

Mardia, K. V., and Zemrock, P. J., 1977, Table of maximum likelihood estimates for the Bingham distribution: Journal of Statistical Computation and Simulation, v. 6, p. 29–34.

Ramsay, J. G., 1958, Superimposed folding at Loch Monar, Inverness-shire and Ross-shire: Geological Society of London Quarterly Journal, v. 113, p. 271–308.

——— 1967, Folding and fracturing of rocks: New York, McGraw-Hill, 568 p.

Ramsay, J. G., and Huber, M. I., 1987, Techniques of modern structural analysis, Part II, Folding and fracturing of rocks: London, Academic Press, 700 p.

Scheidegger, A. E., 1965, On the statistics of the orientation of bedding planes, grain axes and similar sedimentological data: U.S. Geological Survey Professional Paper 525-C, p. C164–C167.

Turner, F. J., and Weiss, L. E., 1963, Structural analysis of metamorphic tectonites: New York, McGraw-Hill, 545 p.

Vollmer, F. W., 1985, A structural study of the Grøvudal fold-nappe, northern Dovrefjell, central Norway [Ph.D. thesis]: Minneapolis, Minnesota, University of Minnesota, 233 p.

——— 1988, A computer model of sheath-nappes formed during crustal shear in the Western Gneiss Region, central Norwegian Caledonides: Journal of Structural Geology, v. 10, p. 735–743.

——— 1989, A triangular fabric plot with applications for structural analysis [abs.]: Eos (American Geophysical Union Transactions), v. 70, p. 463.

Watson, G. S., 1965, Equatorial distributions on a sphere: Biometrica, v. 52, p. 193-201.

——— 1966, The statistics of orientation data: Journal of Geology, v. 74, p. 786–797.

Weiss, L. E., and McIntyre, D. B., 1957, Structural geometry of Dalradian rocks at Loch Leven, Scottish Highlands: Journal of Geology, v. 65, p. 575–602.

Whitten, E.H.T., 1966, Structural geology of folded rocks: Chicago, Illinois, Rand McNally & Co., 663 p.

Williams, P. F., 1985, Multiply deformed terrains—Problems of correlation: Journal of Structural Geology, v. 7, p. 269–280.

Woodcock, N. H., 1977, Specification of fabric shapes using an eigenvalue method: Geological Society of America Bulletin, v. 88, p. 1231-1236.

Manuscript Received by the Society October 27, 1988
Revised Manuscript Received October 14, 1989
Manuscript Accepted November 13, 1989